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LEVEL II

RELATING EXPECTED INVENTORY
BACKORDERS TO SAFETY STOCK
INVESTMENT LEVELS

Dennis M. Carpenter, Captain, USAF

LSSR 91-81

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The objective of the research was to develop a method to define and predict the relationship between inventory performance and safety stock investment for the Defense Electronics Supply Center (DESC) inventory. DESC uses the model prescribed by DoD Instruction 4140.39 to set individual item safety stock levels. This model minimizes the sum of the variable order and holding costs subject to a constraint on the expected inventory performance as measured by the number of time-weighted essentiality-weighted requisitions short (backorders). An important consideration in selecting the constraint for this model is the safety stock investment required for various levels of performance. This thesis uses multi-variate regression analysis and forecasting techniques to predict the relationship between expected performance and required investment. The author concludes that this method produces accurate predictions of the relationship. The recommended model produced average absolute errors of about three percent when tested against historical data from the DESC inventory.

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RELATING EXPECTED INVENTORY BACKORDERS TO SAFETY STOCK INVESTMENT LEVELS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Systems Management

By

Dennis M. Carpenter, BS
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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

CHAPTER I

INTRODUCTION

The basic subject of this thesis is setting optimum safety stock levels in multiple-item vendor inventories under system constraints. An inventory is essentially an idle resource which is being temporarily stored for use at some future time (16:1). The primary purpose for storing resources is to decouple the otherwise dependent functions in the production-distribution-consumption chain (2:389) by separating the supply and demand processes (16:1). Vendor (purchasing) inventories are those which are procured from outside the organization as opposed to production inventories which are produced and stored by the eventual user (2:390). The function, then, of vendor inventories is to separate the supply process involved in procuring the resource from the demand process of consuming the resource or redistributing (selling) the resource outside the organization. Vendor inventories are typically multiple-item inventories in that most organizations store several different resources rather than a single item (2:391).

Because maintaining any type of inventory incurs expenses, the benefits of holding the inventory must exceed the holding costs (16:6). The process of balancing the costs incurred with the costs avoided by holding stocks is inventory control (16:7). As Hadley and Whitin (9:1) state, the two fundamental issues in controlling any inventory are when to order and how much to order. In addition, when the demand for an item is uncertain, the level of safety or buffer stock carried to meet unpredicted demand must be considered when making these two decisions (9:161). The specific subject of this thesis is the problem of determining the optimum item safety stock levels in the Defense Electronics Supply Center (DESC) inventory subject to some constraints. The type of constraints considered are those that place limits on the combined safety stock level of all the items in the DESC inventory.

The background for the research problem is presented in the next section, which defines the operating characteristics of the DESC inventory and describes the organizational setting for the problem.

Background

The DESC Inventory

DESC manages and controls the inventory of non-reparable electronics parts and components for the U.S. Department of Defense (DoD). DESC procures this inventory from numerous sources and then supplies the various DoD components with items as requested (23). Thus, the DESC inventory is a vendor inventory in which items are replenished instantaneously when an order arrives from a supplier. Vendor inventories are normally differentiated from production inventories which are replenished gradually as the items are produced (2:390). Another common distinction made between types of inventories is whether they are static or dynamic. Static inventories are those for which a single procurement is made to satisfy the demand in a single finite time period, while a dynamic inventory has an infinite planning horizon and the inventory is repeatedly restocked (24:6). DESC restocks items as required; therefore, the DESC inventory can be classified as a dynamic vendor inventory.

Inventories may also differ in how often the stock level is checked. Stocks can be reviewed either continuously with fixed-quantity orders placed at any time the reorder point is reached, or they can be reviewed periodically to determine when and how much to order (16:9). Although DESC reviews stock levels at three-day intervals, the inventory is controlled as if it were a continuous review inventory (1). This is reasonable because continuous review can be assumed for practical purposes if the time interval between reviews is negligible compared to the interval between successive stock depletions (16:40). Figure 1 shows the basic behavior for a single item in a continuous review, dynamic, vendor inventory. The inventory level decreases over time at the rate of demand, D, until an order is received. Then the stock is instantaneously replenished by the standard quantity, Q, which was ordered when the inventory level reached the reorder point, R.

The DESC inventory can be further classified as one which allows backorders as opposed to the lost sales type. Inventories which allow backorders eventually satisfy all shortages as stocks are replenished. If backorders are not allowed, any demand that cannot be satisfied from current stocks represents lost sales (2:391). Figure 2 depicts the behavior of an inventory item resulting from allowing backorders in the model shown in Figure 1. In the model with backorders, the order quantity is used both to fill backorders and to replenish the inventory. The advantage of allowing backorders is that fewer orders are placed and inventory levels are lower. Consequently, both the costs of ordering stocks and holding stocks may be reduced (2:401).

The final characteristic to be considered in defining the DESC inventory is whether demands and leadtimes are deterministic or stochastic. The models shown in both Figures 1 and 2 assume that the demand for an item and the delay, or leadtime, between placing an order and receiving the order are known with certainty, and that demand is uniform and continuous with respect to time. However, the demand or leadtime may be variable and unknown and would, therefore, be stochastic rather than deterministic. When the demand or leadtime are stochastic, safety stocks can be maintained to absorb some amount of variation in the demand or leadtime (2:390). In the DESC system, safety stocks must be carried because both the demand and leadtime are stochastic. Figure 3 is a model of this situation. Although backorders are allowed, safety stocks are still necessary to insure excessive backorders do

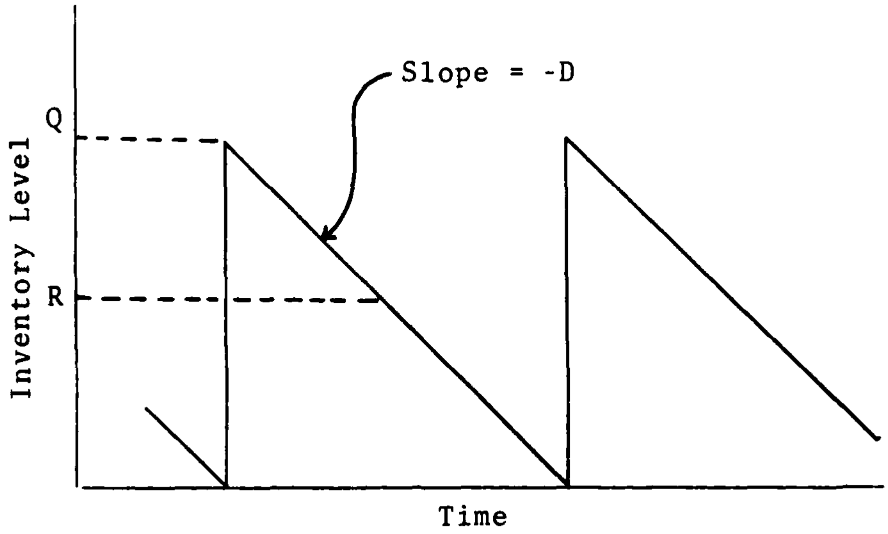


Figure 1

Behavior of a Continuous Review,
Dynamic, Vendor Inventory

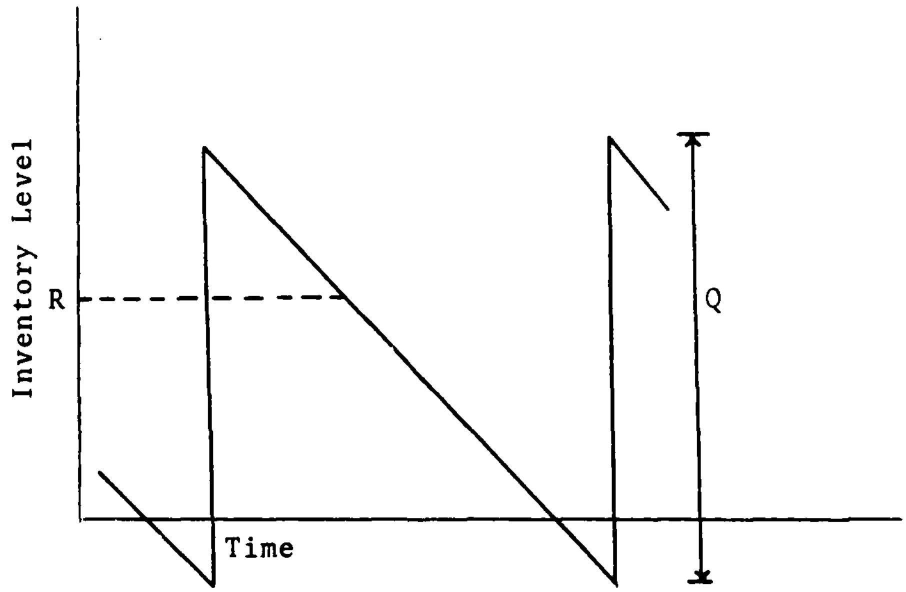


Figure 2

Behavior of a Continuous Review,
Dynamic, Vendor Inventory with
Backorders

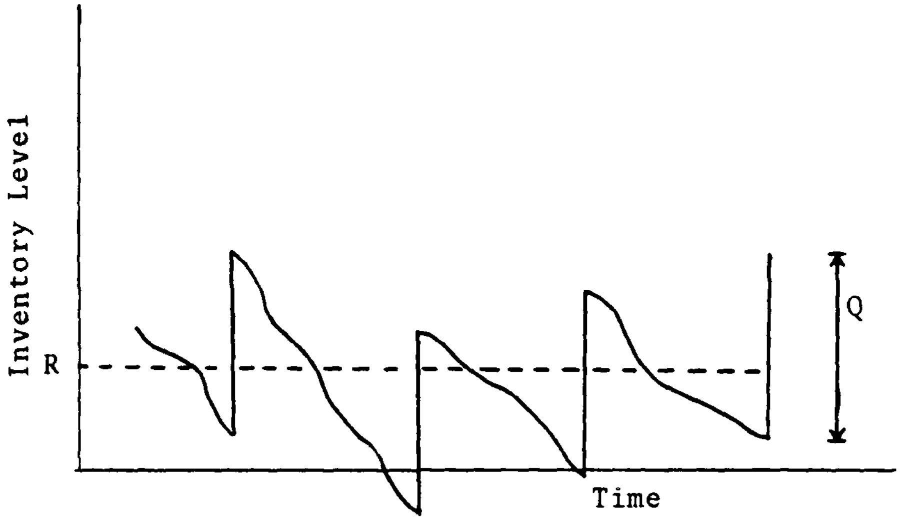


Figure 3

Inventory with Stochastic
Demand and Leadtimes

not accumulate.

Although the DESC inventory system could be defined further, the characteristics described are sufficient to understand the environment in which the optimum safety stock levels must be determined. Backorders are allowed for each of the numerous different items in the inventory. For each item, safety stock can be carried to either avoid shortages or to limit the number of backorders. Thus, if high levels of safety stock are carried for each item, the total number of backorders is reduced and the overall system performance is improved in terms of being able to supply required items to the DoD components. At the same time, higher safety stock levels require a larger inventory investment. The problem, then, is to determine the optimum safety stock level for each inventory item based on the level of system performance which can be achieved with the available funds. The organizational setting of this problem is briefly described in the next section.

DESC Organization

DESC is a component of the Defense Logistics Agency (DLA), a DoD organization responsible for providing services and supplies which are used in common by all the Military Services. DLA was established in 1962 to consolidate several DoD supply agencies which had been managed by separate Military Services. Today, DESC is one of six Defense Supply Centers under the direction of DLA (25:1). DESC is the DoD inventory control point for non-reparable electronics parts and components. DESC is responsible for the supply management of electronics components used by the Military Services, including consolidating requirements, procurement, storage, distribution, and financial accounting. In addition, DESC provides supply support to all Federal civil agencies (27:II-2).

DESC is presently the inventory control point for over 750,000 items which represent an annual inventory investment of over $450 million. This inventory supplies some 22,000 military and civil agency customers (23). In controlling this inventory, DESC is responsible for implementing DoD policy to determine order quantities, reorder points, and other inventory operating rules for each item. Because the supply and demand processes differ greatly amongst the various items, DESC uses different control procedures for different categories of items based on such factors as the demand rate, leadtime, and item cost. Of specific interest here is that some items are controlled using fixed safety stock levels, while others have variable safety levels which are changed periodically based on updated information (1). This study is concerned only with the total backorders and the associated funding requirement for those items which DESC controls by using a variable safety level operating rule.

DLA/DESC Variable Safety Level

The DoD policy and the methods used by DLA and DESC to determine safety stock levels have evolved and changed over the years. This section presents the background, development, and application of the model presently used by DESC and DLA to determine variable safety levels.

DoD Policy

Safety level is defined by DoD as "the quantity of material which is required to be on hand to permit continued operation in the event of minor interruption of normal replenishment or unpredictable fluctuation in demand [28:2]." According to a DLA inventory management specialist (25:1), the only DoD guidance on the subject of safety levels prior to 1970 was DoD Instruction 4140.11, which simply stated that variable safety levels were preferable to fixed levels. Each DoD component was left with the responsibility to establish an appropriate policy and develop mathematical models to determine inventory safety levels. As a result, DoD supply centers were operating under a wide range of safety level policies of varying effectiveness and cost.

In 1970 the DoD published its policy for determining order quantities and safety stock levels for non-reparable items in DoD Instruction 4140.39. This policy specifies both a system objective function which should be optimized and a system constraint which must be met. The objective function is the total variable cost of ordering and holding inventory at the inventory control point. This cost should be minimized subject to a system constraint on the number of requisitions short (backorders) considering the average number of days delay and the essentiality of each item. Thus, the objective of this policy is:

> To minimize the total of variable order and holding costs subject to a constraint on time-weighted, essentiality-weighted requisitions short [28:2].

DoD Instruction 4140.39 also prescribed the general model to be used in implementing the policy statement. The annual variable order cost for each item was given as:

$$OC_i = \frac{AD_i}{Q_i}$$

where

A = order cost

D_i = mean annual demand for the i^{th} item

Q_i = order quantity for the i^{th} item

and the total variable order cost for an N item inventory is:

$$OC = \sum_{i=1}^{N} \frac{AD_i}{Q_i}$$

The holding cost was applied only to the on-hand inventory. It was noted that to be precise, expected backorders (due-ins) should be included, but that "this term has little effect on optimal decision rules [28:Encl 2, p. 1]." Therefore, the average inventory per item was defined as :

$$OH_i = R_i + \frac{Q_i}{2} - \mu_i$$

where

R_i = mean leadtime demands and safety level for the i^{th} item

Q_i = procurement cycle (order quantity) for the i^{th} item

μ_i = mean leadtime demand for the i^{th} item

and the total variable holding cost for the inventory is:

$$HC = \sum_{i=1}^{N} IC_i(R_i + \frac{Q_i}{2} - \mu_i)$$

where

I = holding cost rate

C_i = cost of i^{th} item

The expected essentiality-weighted requisitions short (in terms of orders rather than units) at a random point in time was expressed as:

$$RS = \sum_{i=1}^{N} \frac{E_i}{S_i Q_i} \int_{R_i}^{\infty} (x - R_i)[F(x+Q_i\ ;L) - F(x;L)]dx$$

where

E_i = item essentiality

S_i = average units demanded per requisition

$F(x+Q_i;L)$ = probability that the number of units demanded in leadtime L is $\leq x + Q_i$

$F(x;L)$ = probability that the number of units demanded in leadtime L is $\leq x$.

Finally, the inventory total variable cost to be minimized was defined as:

$$TVC = OC + HC + \lambda RS$$

where λRS is the implied cost of time-weighted shortages. Minimizing this total variable cost is equivalent to using the method of Lagrange multipliers to minimize the sum of the ordering and holding cost subject to a constraint on the expected essentiality-weighted requisitions short (11:552).

The expected backorder model used to determine the implied shortage cost was derived from a model developed by Hadley and Whitin in 1963 (25:3). The implementation of the model, including determination of the shortage parameter (λ), the choice of leadtime demand distributions, and the development of explicit formulas was left to the individual DoD components. The instruction noted that if the shortage parameter is specified in terms of a performance level, this will affect the required funding level and vice versa. Further, it pointed out that the shortage parameter used by the DoD to compute the budget would be a function of performance goals while the parameter used in day-to-day operations might be a function of funding levels or other management decisions (28:Encl 2, p. 3).

The DLA Safety Level Model

In implementing the DoD policy, DLA adopted a modified Wilson EOQ model to determine order quantities and a model developed by Presutti and Trepp to determine safety stock levels (6:A-1). The model chosen to determine item safety levels uses the number of backorders (a performance measure) as the system constraint, and assumes that demands are normally distributed using the following as an approximation to the normal probability density function (18:244):

$$F(x) = \frac{\sqrt{2}}{2\sigma} \exp(-\sqrt{2}\left|\frac{x-\mu}{\sigma}\right|); \text{ for all } x$$

Using a technique developed by Hadley and Whitin (9:178), Presutti and Trepp show (18:246) that, assuming the above function describes the demand distribution, the expected number of units in a backorder status at a random point in time is:

$$B_T = \frac{0.5}{2} \frac{\sigma^2}{Q} (1-\exp(-\sqrt{2}\ Q/\sigma))\ \exp(-\sqrt{2}k)$$

Presutti and Trepp then use this expression to develop four inventory models which differ in their treatment of backorder penalties and holding costs. DLA chose to adopt the model in which the backorder penalty is time-weighted and the holding cost is applied to the inventory position (on-hand plus due-ins) rather than the on-hand inventory (6:A-2) used in the DoD model.

The basic DLA model, using DLA notation is (6:A-2):

$$\text{minimize} \sum_{i=1}^{N} \frac{P_i AD_i}{Q_i} + \sum_{i=1}^{N} a_i C_i (u_i + k_i \sigma_i + \frac{Q_i}{2})$$

subject to

$$\sum_{i=1}^{N} \frac{.5}{2} \frac{Z_i \sigma_i^2}{S_i Q_i} (1-\exp(-\sqrt{2}\frac{Q_i}{\sigma_i}))\ \exp(-\sqrt{2}\ k_i) \leq \beta$$

where

k_i = safety level factor

Q_i = Economic Order Quantity

μ_i = leadtime demand or expected due-ins (3:187)

a_i = holding cost rate

C_i = unit price

σ_i = standard deviation of leadtime demand

Z_i = essentiality factor

S_i = average requisition size

AD_i = annual demand in units

P_i = procurement (order) cost

β = number of backorders at a random point in time

Consistent with the DoD model, this model minimizes total ordering and holding costs subject to a constraint on the number of backorders. The Method of Lagrange then yields the following formula to compute item safety level factors:

$$k_i = -\frac{1}{\sqrt{2}} \ln[\frac{S_i\sqrt{2}\ Q_i a_i C_i}{.5(-\lambda) Z_i \sigma_i (1-\exp(-\sqrt{2}\ \frac{Q_i}{\sigma_i}))}]$$

where

$$-\lambda = \sum_{i=1}^{N} \frac{\sigma_i a_i C_i}{\sqrt{2}\beta}$$

DLA uses an approximation for the standard deviation of lead-time demand, which is 1.25 times the mean absolute deviation of demand forecast errors over the leadtime (σ_i = 1.25 MADLT) and a constant holding rate ($a_i = a$). Using these values and substituting the expression for λ into k_i yields:

$$k_i = -\frac{1}{\sqrt{2}} \ln\left[\frac{2.56\ S_i Q_i C_i \beta}{Z_i\ \mathrm{MADLT}_i (\Sigma C_i\ \mathrm{MADLT}_i)(1-\exp(-\sqrt{2}Q_i/1.25\ \mathrm{MADLT}_i))}\right]$$

The variable safety level for an item is then calculated as

$$\mathrm{VSL}_i = 1.25\ k_i\ \mathrm{MADLT}_i$$

which is the safety level factor times the approximation for the standard deviation of the leadtime demand.

Model Application

DESC has used the above model to calculate variable safety levels for about 150,000 inventory items each quarter since January 1976. Other items, such as new items with no demand history or items requisitioned one time, are managed using various other operating rules. From January 1976 through December 1980, the average investment in variable safety level stock was about $53 million, and the standard error from the desired funding level was about $5.9 million. During this period, the method used to include the desired funding level as a constraint on safety stock levels was to use experience to select a backorder constraint which would result in the desired safety level dollars after the item safety levels were calculated according to the DLA model. Thus, the objective in selecting a backorder constraint as an input to the DLA model has been to choose a value which sets item safety levels such that the total required safety stock investment equals the desired funding level (1).

The selected backorder constraint was then input to a computer program which calculates the item variable safety levels based on the DLA model. This same program also accomplishes the inventory quarterly update, calculates numerous other item values, and after running about 18 hours, produces a new inventory policy to be used in the upcoming quarter for each item. In addition to computing the safety level dollars required for the input backorder constraint, the program also computes five other funding levels based on different (selected) backorder constraints. The performance values (backorders) are then plotted against the funding levels (safety level dollars) to derive a curve which shows the correct backorder constraint which should have been used to achieve the funding target. However, to use the correct constraint for the upcoming quarter would require rerunning the complete quarterly update program which would, of course, be extremely costly due to the amount of computer time required. Therefore, the post-analysis to determine the correct backorder constraint has been of little practical use (1).

Problem Statement

At the present time, the model used by DESC, and all other DLA supply centers, to determine variable safety levels does not explicitly consider any constraint on funds available for the inventory safety stock. The funding constraint is implicitly included in the model by selecting a backorder level which will maximize system performance while satisfying the funding constraint. Using experience to select the backorder constraint has produced relatively large errors between the target funding level and the required safety level dollars. This causes the individual item variable safety levels for a particular quarter to be set at a level which is significantly different from the optimum level considering the funds available. Thus, the basic problem of the current model is that it is not effective in establishing safety levels that are consistent with funding targets. DESC is interested in resolving this problem to gain better control of the system and improve service (1).

Research Objectives

Although DESC inventory managers are concerned about maintaining certain performance levels, the primary constraint in establishing item safety levels is the funds available for safety stock. Therefore, the overall objective of the research was to develop an efficient and accurate method to determine item safety levels subject to a budget constraint. However, since an accurate prediction of system performance is important and can be used to demonstrate the effects of different funding levels, another goal was to determine the relationship between performance levels and funding requirements. In fact, if this relationship is established prior to running the quarterly update, then it will make no difference whether the system constraint used in the model is in terms of performance level or funding level. The primary objective of the research, then, was to develop a method to define the relationship, at a specific point in time, between the expected number of total backorders and the required funding level. This information can then be used to set item safety levels based on the level of system performance which can be achieved with the available funds.

Summary

This chapter introduced the research problem. After some preliminary comments about inventories in general, the DESC inventory system was defined as a continuous review, dynamic, vendor inventory system which allows backorders and is subject to stochastic demands and leadtimes. As a part of the Defense Logistics Agency organization, DESC uses the DLA prescribed model to determine item variable safety levels. The objective of this model is to minimize the total system costs of ordering and holding the inventory subject to a constraint on the total number of backorders allowed. The problem being experienced with this model is that it does not consider the amount of funds available for safety stock, which causes large errors between the required safety level investment and the funding target.

CHAPTER 2

PROBLEM DISCUSSION

The research problem, as introduced in Chapter 1, is to determine the relationship between total inventory backorders and total safety stock investment when the variable safety stock levels are set using the current DLA/DESC model. This chapter examines the problem, primarily from a theoretical viewpoint, to establish the basis for a solution method. The purpose is to clearly define the research problem within the framework of generally accepted inventory principles, the objectives of the organization, and the alternatives available to solve or alleviate the problem. The first section presents a background for the problem by identifying the inventory characteristics which lead to multiple management objectives, the nature of the relationships that exist between the objectives, and the performance measures used to evaluate multiple-item inventory objectives. Next, the theory supporting the DLA model is examined to identify the strengths and weaknesses of the model and the source of the research problem. The third section discusses the problem of finding the costs for maintaining different levels of backorders and presents the basic alternatives available to overcome the problem. Finally, the last section briefly presents some alternative models for calculating safety stock levels which might provide a solution to the problem if DESC were willing to adopt a new model.

Safety Stock Management Objectives

As suggested in Chapter 1, inventory control is concerned with balancing the costs and the benefits of holding stocks. This balance is directly affected by the level of stock held by the organization (14:111). The effects of low stock levels are that customer demands often cannot be satisfied, the number of stock replenishment orders is high, many orders must be expedited, and there are few opportunities for quantity discounts. Thus, low stock levels cause low service levels and high ordering costs. High stock levels will increase the service level and decrease ordering costs. However, high stock levels also have some undesirable effects such as high storage costs, high capital investment and increased risk of obsolescence. Thus, high stock levels increase holding costs. The relationships between these variables are that when stock levels and service levels increase, holding costs increase and ordering costs decrease.

There is also some inventory level at which the total costs of ordering and holding the inventory are minimized (9:35). If the only objective of inventory management were to minimize the ordering and holding costs and these costs could be quantified, the appropriate inventory level could be found by minimizing the sum of the ordering cost function and the holding cost function. This inventory level would, in turn, determine the level of service. The problem with this approach is that not all ordering and holding costs can be quantified and the costs of inventory shortages must be considered. Therefore, most organizations have at least two other inventory management objectives in addition to minimizing the quantifiable ordering and holding costs (22:39). One of the other objectives is to improve the level of service or provide a minimum level of service. The second objective is to reduce the size of the inventory or to limit the size to some desired level. One reason for these two additional objectives is that the associated costs are difficult, if not impossible, to measure directly (16:22). Service level is actually a surrogate measure for shortage costs, while inventory size can be considered as a substitute for the opportunity cost of the inventory investment, which is one of the holding costs. These costs are always difficult to measure, and in the case of non-profit-seeking organizations such as the DoD, it is often not possible to quantify shortage costs and opportunity costs (31: Chap. 7). Thus, the basic objectives of inventory management are to minimize the quantifiable ordering and holding costs, provide some level of service, and limit the size of the inventory.

One of the difficulties in inventory management is caused by the fact that most inventories are multiple-item and there are interactions among the items. There are many types of possible interactions between the items. For example, the items may be substitutes for each other; ordering costs might be reduced by ordering several different items from the same supplier; or the items may be competing for restricted resources such as warehouse space or inventory investment dollars (9:54). Interaction between the many inventory items requires the use of aggregate performance indices and more complex models. Aggregate indices of costs, service levels, and inventory size are necessary to measure the effectiveness of meeting the objectives for the total inventory (29:8). More complex models are required to reflect the interactions among the many different items (16:112).

Another management problem is selecting the appropriate indices and establishing specific inventory objectives in terms of the selected indices. There are numerous indices in common use that provide aggregate measures of service level or inventory size. Some of these are (29:8):

1. Value of inventory on hand
2. Value of inventory on order
3. Value of inventory on hand and on order
4. Number of orders placed
5. Number of backorders

After selecting indices which are appropriate for the particular organization, the management objectives must be specified in terms of the indices. However, the ordering and holding costs, service level, and inventory size are not independent of each other. Because these objectives are interrelated, they must be considered simultaneously and tradeoffs made to achieve a balance which is considered optimal for the particular organization in question.

The selected balance between the desired objectives may not be achieved because of the stochastic processes inherent in many inventory systems. The distinguishing characteristic of stochastic inventories is that stocks are not ordered to meet a particular known demand (16:56). Instead, stocks are ordered based on the current inventory status and the knowledge of past demands and leadtimes with uncertain future demands and leadtimes. Because the demand and leadtime for single items is uncertain, the inventory size, service level, and costs are probabilistic and can be viewed as random variables. Indices which are based on many items with probabilistic demand or leadtime must be described stochastically with the leadtime and demand pattern of the individual items affecting the probability distribution of the indices (29:8). Because the indices of stochastic inventories are random variables, the established objectives must be viewed in terms of expected values which are unlikely to be exactly met.

Inventories with stochastic demands and leadtimes also require safety stock to reduce the possibility of stockouts caused by leadtime demand forecast errors. Safety stocks can be defined as the difference between the expected demand for a period and the level of stocks held to meet demand for the period (31:42). In the continuous review situation, the stocks are ordered when the inventory level reaches the reorder point. The difference between the reorder point and the safety stock level is the expected or mean demand during the leadtime before the next order is received. The safety stock, then, is used to meet demands in excess of the expected demand. This excess demand represents the forecast error of demand during the leadtime period. Because the demand forecast error may exceed the level of safety stocks held, stock-outs may occur and backorders will accumulate.

As with the overall inventory, optimum safety stock levels are determined by consideration of the costs and benefits of stocking various levels. The same basic relationships exist. As the safety stock level is increased, the level of service increases, holding costs increase, and ordering costs may decrease (31:42). At some point, an optimum balance exists between the desired stock level, service level, and costs. In multiple-item inventories, these effectiveness measures can again be represented by various aggregate indices to indicate the performance over all items. These safety stock indices are, of course, based on single items with stochastic leadtime demands; therefore, the indices are stochastic and can be represented as probability distributions which are affected by the leadtime demand patterns of the individual items. The safety stock policy for a multiple-item inventory represents an attempt to find an optimum balance between the applicable costs, safety stock level, and service level as measured by stochastic aggregate indices. The current DESC safety stock policy is determined by the DLA model which is discussed in the next section.

Analysis of the DLA Model

The DLA model currently used by DESC to compute variable safety stock levels stems from techniques and general models developed by Hadley and Whitin for continuous review inventories with stochastic demands. In developing their stochastic models, Hadley and Whitin assumed that "the process generating the demands does not change with time" (i.e., the mean rate of demand is constant over time) and used the minimization of variable costs as the criterion to determine the optimal inventory policy. They also caution that, by definition, a continuous review model assumes that an order is placed precisely when the reorder point is reached. This, in turn, implies that the number of units demanded per requisition cannot be a random variable because then it would be possible to overshoot the reorder point before an order could be placed. Therefore, in cases where the units per requisition is a random variable, it is inappropriate to order a fixed quantity each time an order is placed. Hadley and Whitin note that for practical purposes continuous review models can be used for inventories with a variable requisition size, provided the probability of overshooting the reorder point is very small. In such cases, the optimal reorder point R and order quantity Q can be found with a continuous review model, then the actual order quantity is Q plus the amount by which the reorder point was exceeded (9:Sec 4-1). This is the method used by DESC.

The DLA continuous review model is based upon the development by Hadley and Whitin of a model which assumes that item leadtime demand is approximately normally distributed. They note that for large leadtime demands, discreteness can be ignored and all variables can be treated as continuous. In addition, the continuous normal distribution will be a good approximation for the actual demand distribution when the demand is sufficiently large. The model also assumes that procurement leadtimes are constant, and that the entire order is received at the end of the leadtime. The model allows backorders and uses the inventory position, rather than the inventory on hand, to define the reorder point. The objective of the Hadley and Whitin model is to develop an expression for the average annual cost of the inventory which should be minimized to find the optimal order quantity and reorder point. However, in the course of developing the cost function, it was also necessary to determine the expected number of units in a backorder status at a random point in time, given the assumptions of the model (9: Sec 4-9). It is this expression for expected backorders that was used in developing the DLA model (18:246).

In their development of stochastic models, Hadley and Whitin also address the difficulty in determining the cost of a backorder and produce a model which does not require that explicit backorder costs be assigned. The objective of this model is to minimize the average annual ordering and holding costs subject to a constraint on the expected number of units in a backorder status at any point in time. Using the theory of Lagrange multipliers, Hadley and Whitin show that this is equivalent to minimizing (in simplified form):

$$TVC = OC + HC + \lambda B$$

where

TVC = total variable cost

OC = ordering cost

HC = holding cost

B = expected number of backorders

λ = Lagrange multiplier

This general model will determine the optimum order quantity and reorder point for an item and uniquely determine λ, which is the implicit cost of a backorder. Thus, by specifying a backorder constraint, a unique implicit backorder cost is determined (9: Sec 4-16).

This is precisely the same model, presented in Chapter 1, which was adopted as the DoD policy for determining order quantities and safety stock levels. The specific ordering and holding cost functions used in the DoD model are also exactly the same functions developed in the Hadley and Whitin model (9:219). In addition, although the DoD model does not specify a particular demand distribution, the expression given to determine requisitions short is simply a generalized form of the expected backorders expression developed by Hadley and Whitin for their model with normally distributed demands (9:193). The only change made by the DoD in adopting the

Hadley and Whitin model was to multiply the expression for expected backorders by E/S (see page 14), which has the effect of weighting the backorders with an essentiality factor and converting the number of units backordered to the number of requisitions backordered. Because the DoD model is the same model as developed by Hadley and Whitin, the same assumptions must apply, although none are specified for the model presented in DoD Instruction 4140.39. Both models also share a computational difficulty in that they require evaluation of a complex integral to determine the number of expected backorders.

The more specific DLA model basically makes two extensions to the DoD model. First, in developing the model used by DLA, Presutti and Trepp substituted for the normal distribution a probability density function which approximates the normal distribution, but can be easily integrated. This eliminated the computational problem of the DoD model by allowing the expected number of backorders to be explicitly defined as an easily calculated expression rather than in terms of a complex integral. Secondly, Presutti and Trepp explicitly define the safety stock level in terms of the standard deviation of the leadtime demand multiplied by some safety factor. These two extensions to the DoD model allow the safety factor and, therefore, the safety level to be explicitly defined by an easily calculated expression. The assumptions upon which Presutti and Trepp based the development of the DLA model are the same as those used by Hadley and Whitin. In addition, of course, they assumed that the substitute probability density function is an adequate approximation for the normal distribution over the range of interest (18).

In using the model to calculate safety levels, DLA also assumes that the standard deviation of the leadtime demand equals 1.25 times the mean absolute deviation of the leadtime demand forecast errors. This assumption is valid when the forecast errors are normally distributed (4:242). DLA also uses a constant holding cost rate for all inventory items. The current model used by DESC, then, is based on several important assumptions which can be summarized as follows:

1. The mean rate of demand for each item remains constant over time.
2. The number of units per requisition is not a random variable.
3. The procurement leadtime for each item is constant.
4. The entire order is received at the end of the leadtime.
5. Leadtime demands are large enough for the demands to be approximately normally distributed.
6. The substitute function adequately approximates the normal distribution over the relevant range.
7. The demand forecast errors are normally distributed.
8. The holding cost rate is constant for all items.

It is difficult to determine the validity of these and other assumptions which may be implicit in the model or to determine the aggregate effect of any deviations. In addition to the question of whether it is a reasonably accurate representation of the DESC inventory, the model has some characteristics which should be noted. First, the model allows the customer service level of the inventory to be managed as an aggregate index which is measured by the total number of requisitions short. Also, the order quantity in this model is completely independent of the backorder constraint which allows the level of customer service to be adjusted by changing only the safety stock levels and using the same economic order quantities (18:249). In addition, for a given service level, or backorder constraint, and any set of order quantities, this model will minimize the cost of holding the safety stock (18:250). However, the model may not allocate adequate safety stock levels to high cost items. Since it is less expensive to procure lower cost items to reduce the number of backorders for the total inventory, high cost items will have lower safety levels. Finally, use of the model implicitly assumes that enough funds are available to purchase the levels of safety stock determined by the backorder constraint. When funds are restricted, it is necessary to be able to select the backorder constraint on the basis of the available funds.

Relating Safety Stock Level to Service Level

In discussing restricted funds, Hadley and Whitin note: "None of the inventory models discussed in operations research appears to provide an adequate treatment of budget constraint [10:152]." Like most models, the DLA model balances the inventory operating costs with the service level without considering the stock levels as represented by the budget constraint. In the DLA model, the service level, measured as the number of requisitions short, is determined solely by the safety stock level, which is measured in dollars invested in safety stock. Therefore, when the inventory managers have limited funds to invest in safety stock or want to balance investments against service, they must be able to determine the variable safety level dollars (VSL$) required for any specified level of backorders (β). In establishing the model to be used to calculate safety levels, DoD Instruction 4140.39 notes that the shortage parameter λ can be selected to both control the safety level and to satisfy other constraints such as the funding level (28:2). This implies that λ might be used to relate VSL$ to β and select an appropriate balance; however, no attempt was made to define this relationship.

Several studies concerned with the model (3, 5, 7, 15) also note that λ can be set to control β and/or VSL$. However, again no method is suggested about how λ could be used as a control or how a relationship between β and VSL$ might be established using λ. Deemer emphasizes that λ "is only an implied shortage cost, the true cost being unknown [5:7]." Therefore, a value of λ has little meaning in itself and is simply a number which has resulted from inventory policy decisions. Since λ is clearly a function of β (see page 14), and β is determined by VSL\$, λ could possibly also be defined as a function of VSL\$. These two functions could then be equated to possibly derive an explicit expression relating VSL\$ to β. However, it makes little sense to complicate the problem by including an implied cost rather than attempting to find a direct relationship between VSL\$ and β.

Presutti and Trepp suggest that different values of λ be used to calculate the values for β and VSL\$ from the current inventory data, then to plot the values to obtain two curves (18:250). One curve will relate λ to service level and the other to investment level. This procedure has the disadvantage, among others to be discussed shortly, that it also unnecessarily relates β and VSL\$ to λ rather than directly to each other. Another indirect approach, used by the Navy on a different form of the DoD model, is to calculate service levels based on changing values of λ and iterate to a solution which satisfies the funding constraint (15:21). Although this technique produces a solution which satisfies the funding constraint, the constraint is necessarily somewhat arbitrary since the service level is not known until after inventory policy is calculated. Both this procedure and the one suggested by Presutti and Trepp are also undesirable because they require that several values of each variable be calculated. This is necessary to obtain a reasonably accurate curve or to iterate to a reasonably close solution and increases the cost of the analysis.

DLA suggests the direct approach that only VSL\$ and

β be computed and then plotted against each other to derive a curve as shown in Figure 4 (6:3). The basic form of this curve is generally known (4:20; 7:III-17) and easily explained. At zero VSL$, or no safety stock, there is some maximum number of expected backorders. As VSL$ increases, backorders decrease at a diminishing rate until eventually additional safety stock has little effect on decreasing the number of backorders. This approach also requires that several values of both variables be calculated to obtain an accurate curve. In addition, a problem shared by each of these procedures is that the relationships between the variables are probably not stationary and are valid only for the period in which they are computed. Since demands and prices are known to change over time (1), and these effect β, λ, and VSL$, the relationship between β and VSL$ changes over time. Therefore, to obtain the correct curve requires that β and VSL$ be recomputed from the current inventory data each time (at least quarterly) that the relationship is required to make a decision. Due to the importance of being able to select β based on available VSL$, DESC has recently decided to use the DLA suggested approach and calculates the backorders and VSL$, based on the total inventory, prior to each quarterly update.

The problems with these procedures are that they do not provide the information required by DESC, and presumably other DoD supply organizations, or they are very costly because they require many calculations on a large data base. Since the information required is the current relationship

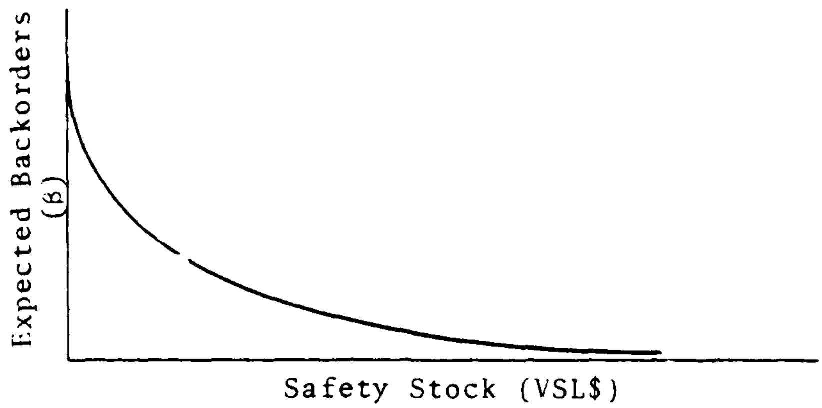


Figure 4

Relationship Between Expected Backorders
and Safety Stock Investment

between β and VSL$, the approach suggested by DLA of deriving the curve defining this relationship certainly provides the information in the most desirable form. Also, although this curve is not stationary and must be derived for each period, it is possible that the basic form of the curve is the same for all periods. This is suggested by the rationale of diminishing marginal returns, which explains the form of the curve, and the fact that the same model is used to derive the curve for each period. If the basic shape of the curve is the same for each period, it should be possible to derive a general equation which defines the basic relationship between β and VSL$. With such an equation, fewer calculations should be required to define the curve. The calculations required to find the equation exactly will, of course, be determined by the number of unknown variables in the equation. However, the apparently simple form of the curve suggests that a relatively simple equation can be devised which would significantly reduce the computational burden of finding the curve. One of the research objectives was to derive such an equation.

Even with a general equation to define the basic form of the curve, the present method would require using current data from all inventory items to find the curve for the current period. To further reduce the cost of finding the curve, it is necessary to eliminate or reduce the amount of data required. One method to do this is to predict the relationship from past aggregate indices which are related to β and VSL\$. DESC calculates several aggregate indices each quarter to measure various aspects of the total inventory. Since β and VSL\$ are closely related to some of these indices, it should be possible to predict the relationship between β and VSL\$, if the other indices can be accurately forecasted. This procedure would completely eliminate the need for data on individual items and greatly reduce the number of calculations required. Another method to reduce the number of calculations is to use a sample of the current data rather than data for all items. Thus, another research objective was to determine how accurately the relationship between β and VSL\$ could be predicted using less data. The method tested was to predict the relationship based on forecasts of the aggregate indices.

Some Alternative Models

Although the purpose of this research was to find a possible solution in the context of the current DLA model, it must be acknowledged that a better solution might be obtained by changing to a different model. Recognizing that most inventory models are "severe abstractions of the real world [3:2]," other models may set safety levels as close to the optimum as the current model and at the same time eliminate or reduce the problem of relating β to VSL$. The current DLA model is one of a class of literally hundreds of traditional cost models which, based on various assumptions, attempt to minimize the sum of holding, ordering, and stockout costs. Even within this class, it is questionable that the DLA model is the most appropriate for the situation. In fact, in discussing a case where this type of model was applied to "a military supply system concerned with stockage of electronic components," Hadley and Whitin concluded that their model was "entirely inappropriate. . .[in] a situation where there was a fixed annual procurement budget [9:403]." A variation of the conventional model which may be more appropriate in this situation is to minimize backorders subject to a constraint on inventory investment. Detailed models of this form have been developed (20, 26) and are a possible alternative to the current model.

Another type of model which may be an improvement over the DLA model is some form of dynamic programming model. The advantage of this class of models is that they take into consideration the changes which occur in inventory systems over time. In particular, dynamic models are helpful when demand distributions, prices, and the items carried in the inventory are changing (9:323). Since these do change in the

DESC inventory, a dynamic model may provide more appropriate safety levels and allow the relationship between β and VSL$ to be determined more accurately. An important disadvantage of dynamic models is that they are generally much more complex and, therefore, more costly to use than steady-state models. One study which developed a dynamic programming model that minimizes expected shortages subject to an investment constraint concluded that the model was computationally impractical for inventories as large as 100,000 items. However, the study suggested that if items could be grouped according to demand rates and costs, the model could be used to allocate shortages and funds to the groups (12:32).

Another alternative is to use a model based upon marginal returns. As noted above, the basic relationship between β and VSL$ is one of decreasing marginal returns. As the number of backorders decreases, the ratio of β/VSL$ increases. A model based on marginal returns iteratively allocates funds to the item which maximizes the desired performance measure per dollar invested (19:3). In the case of backorders, safety level dollars are allocated iteratively to the item with the maximum β/VSL$ ratio. The allocation of VSL$ continues until the investment constraint is reached or an acceptable backorder level is reached. A multi-echelon model of this form, based on minimizing backorders with an investment constraint, was developed for the Air Force base-depot supply system (21). Application of a marginal return model to base level consumable secondary items has also been studied (19). It appears that this form of model has not yet been applied to any problem as large as the DESC inventory, perhaps because of the large number of computations required for so many items. This problem, as with the dynamic programming model, might be solved by grouping items and allocating VSL$ among the groups.

One more approach to the problem involves using the current model to determine an initial inventory policy and another model to alter the policy if sufficient VSL$ are not available. A model has been developed for the Army which modifies inventory policies when insufficient funds are available to buy the levels of stock indicated by the current policy (13:4). This model evaluates the changes in required investment if a particular supply modification, such as cutting reorder points and raising backorders, is implemented. After evaluating several possible modifications, the policy is chosen which meets the funding constraint and provides the highest level of service. While this model was developed to alleviate a problem essentially the same as the one considered here, it appears to be an unnecessarily indirect and expensive approach.

Summary

The purpose of this chapter was to thoroughly review the research problem and to develop and support the specific research objectives. After discussing the basic theory of managing safety stocks in multiple-item stochastic inventories, the theory underlying the DLA model was reviewed. The principle assumptions upon which the model is based were emphasized and some of the more important strengths and weaknesses of the model were noted. Several possible methods to relate backorders to safety level dollars were discussed before presenting the approach selected for this research. This approach was to attempt two methods to reduce the computational effort required to derive a relationship between backorders and safety level dollars. The first method was to derive a general equation which defined the basic form of the β - VSL$ curve. The second method was to reduce the amount of data required by using aggregate indices from past periods to estimate the current relationship between β and VSL$. Finally, some alternative models were presented which showed that there are other approaches to the problem which could be pursued.

CHAPTER 3

METHODOLOGY

The methodology adopted for this research is based on the assumption that a causal relationship exists between VSL\$ and β, as explained in Chapter 2, and that the relationship can be predicted for future periods. In addition, it was assumed that the relationship during any period would be a function of the state of the total inventory as measured by various aggregate indices. The rationale for this assumption is that individual item variable safety levels (VSL_i) are determined by both β and various item characteristics such as price, requisition size, and leadtime demand variance. Therefore, the aggregate safety level measure $VSL\$ = \Sigma VSL_i C_i$ should be a function of both β and some aggregate indices of the item characteristics. Based on these assumptions, the overall approach was to first develop models which would define VSL\$ as a function of β and other available indices for any period. Then, to determine the relationship between VSL\$ and β for the upcoming period, the values of the indices used in the models would be forecasted. The methods used to develop the models and forecast the values of the indices are explained in this chapter. This is preceded by a discussion of the data used and followed by the method used to test the models and forecasting technique.

Data Selection and Definitions

Twenty-eight sets of data were available from DESC covering the period since the DLA model was implemented. Each set contains the aggregate indices produced from quarterly updates which set new safety levels for all inventory items based on the characteristics of the items in the inventory at the time. However, some quarters were represented by two sets of data because the quarterly update had to be rerun. This was necessary either because some input data was incorrect or because the value of β chosen for the initial run produced an unacceptable VSL$ level. In these cases, only the final data set, which established the safety levels actually used for the quarter, were used in the analysis. In addition, the sets from the last two periods available were reserved to test the predictive ability of the models developed. The model development and analysis, then, was based on 21 sets of historical data representing the quarters from January 1976 to December 1980. This data and the data for the two quarters used to test the models are presented in Appendix A. The variable names used for the indices and their definitions are given below:

β - The expected (average) number of time-weighted requisitions short at any point in time

SC - Referred to as the system constant, this is the sum, over all items, of the unit price times the mean absolute deviation of the lead-time demand forecast error ($\Sigma C_i MADLT_i$)

OLD SC - The system constant from the period immediately preceding the period in question

VSL$ - Variable safety level dollars is the sum of the dollar value of the item safety levels ($\Sigma VSL_i C_i$)

DEMAND$ - The value of the forecasted inventory annual demand ($\Sigma 4QFD_i C_i$)

ITEMS - The number of items

0-ITEMS - The number of items with a safety level of zero

FREQ - The annual demand frequency is the total number of requisitions per year for all items

Model Development

Correlation and regression analysis were the basic methods used to develop and compare possible models. Correlation coefficients were used to measure the strength of the pairwise relationships between VSL$, β, and the other available indices. The correlation statistics used were the Pearson product-moment correlation coefficient r and a significance test on r using the Student's t with N-2 degrees of freedom for the quantity (17:281):

$$r \frac{\sqrt{N-2}}{\sqrt{1-r^2}}$$

This information was then used to help determine which variables might be added or deleted in a model to improve the model. Multiple regression analysis was used to develop models and was the primary method used to compare the models. The DLA model was also examined and manipulated to determine what forms of regression models might be suggested by the structure of the relationships between the variables in the DLA model. Finally, the basic VSL\$ - β curve suggested a model form which was then further developed through regression analysis. The model development process eventually produced three basic forms of models, which were compared to select the one which most accurately defined the relationship between VSL\$ and β. The models which were developed are presented below.

Linear Multiple Regressions

Two models were developed using simple linear multiple regression techniques. In one model, VSL\$ was the independent variable, and in the second β was the independent variable. These models were developed using the stepwise regression procedure from the Statistical Package for the Social Sciences (SPSS) (17). In this procedure, one variable is chosen at each step to be added or removed from the regression equation. The selection process is based on the partial F-value of each variable which indicates the amount each variable contributes to reducing the residual sum of squares. If the F-value of a variable falls below a specified value during the stepwise procedure, it is removed from the equation in the next step. If no variables are to be removed in a step, the variable with the highest F-value and not already in the equation is entered. This procedure continues until all the variables not in the equation have F-values below the specified value. The F-value specified to enter and remove variables from the equation was 2.0 for all of the SPSS regressions performed in this study. The form of the two linear models which will be referred to as Models 1 and 2 was:

Model 1: $VSL\$ = C_0 + C_1\beta + C_2X_1 + C_3X_2 + \ldots + C_iX_j$

Model 2: $\beta = C_0 + C_1 VSL\$ + C_2X_1 + C_3X + \ldots + C_iX_j$

where the X_j are the inventory indices and the C_i are the regression coefficients.

Regressions Based on the DLA Model

The second form of regression model tested was derived from the DLA variable safety level model. The objective in developing this form of model was to rearrange and, where possible, simplify the DLA model to obtain an expression which represented a linear or nonlinear regression model with the available indices as the variables. The dependent variable in the model is VSL$ defined as:

$$VSL\$ = \Sigma\, VSL_i C_i$$

or

$$VSL\$ = \Sigma\, 1.25\, k_i MADLT_i C_i$$

The expression for k_i (see page 14) can be rewritten as:

$$k_i = -\frac{1}{\sqrt{2}} \ln \left[\frac{S_iQ_iC_i}{Z_i MADLT_i (1-\exp(-\sqrt{2}Q_i/1.25\ MADLT_i))} \cdot \frac{2.56\ \beta}{\Sigma\ C_i MADLT_i}\right]$$

or

$$k_i = -\frac{1}{\sqrt{2}} \ln \left[\frac{S_i Q_i C_i}{Z_i \text{MADLT}_i (1-\exp(-\sqrt{2} Q_i / 1.25 \text{ MADLT}_i))}\right]$$

$$-\frac{1}{\sqrt{2}} \ln(2.56\beta) + \frac{1}{\sqrt{2}} \ln(\Sigma\ C_i \text{MADLT}_i)$$

This expression of k_i isolates the aggregate indices from the i^{th} item characteristics. If X is substituted for the term in brackets containing all the item characteristics and the resulting expression is substituted for k_i, then,

$$\text{VSL\$} = 1.25\ \Sigma\ \text{MADLT}_i C_i (-\frac{1}{\sqrt{2}})\ \ln(X)$$

$$+\ 1.25\ \Sigma\ \text{MADLT}_i C_i (-\frac{1}{\sqrt{2}})\ \ln(2.56\beta)$$

$$+\ 1.25\ \Sigma\ \text{MADLT}_i C_i (-\frac{1}{\sqrt{2}})\ \ln(\Sigma\ \text{MADLT}_i C_i)$$

The first term in this expression is not represented by any available index, but is a constant, C_0, in any period. Grouping the constants in the other two terms and substituting SC for Σ $\text{MADLT}_i C_i$ yields the model:

Model 3: $\text{VSL\$} = C_0 + C_1 \text{SC} \ln(\beta) + C_2 \text{SC} \ln(\text{SC})$

Because DESC uses the system constant from the previous period instead of the last SC in this expression to calculate safety levels, another model of this form was tested.

Model 4: $\text{VSL\$} = C_0 + C_1 \text{SC} \ln(\beta) + C_2 \text{SC} \ln(\text{OLD SC})$

The third model developed in this group is:

Model 5: $VSL\$ = C_0 + C_1(OLD\ SC)\ \ln(\beta) + C_2(OLD\ SC)\ \ln(OLD\ SC)$

This model was tested to determine how well the old system constant alone could predict the VSL$ - β relationship.

Models 3, 4, and 5 represent nonlinear relationships. However, they are in the form of a linear regression model when the variables in each of the second and third terms are combined and viewed as a single regression variable. Therefore, the SPSS linear stepwise procedure was also used to test these models.

Model Based on the VSL$ - β Curve

The development of this model began by plotting the six VSL$ and β values, provided by each quarterly update, for several periods to determine whether they actually fit the curve hypothesized in Chapter 2. Although the values available were in a relatively small range, they appeared to fit the form of the theoretical curve as shown by the examples presented in Figure 5 . The form of this curve suggested that the nonlinear VSL$ - β relationship for a single period took the form of a power function or logarithmic function. One form considered was

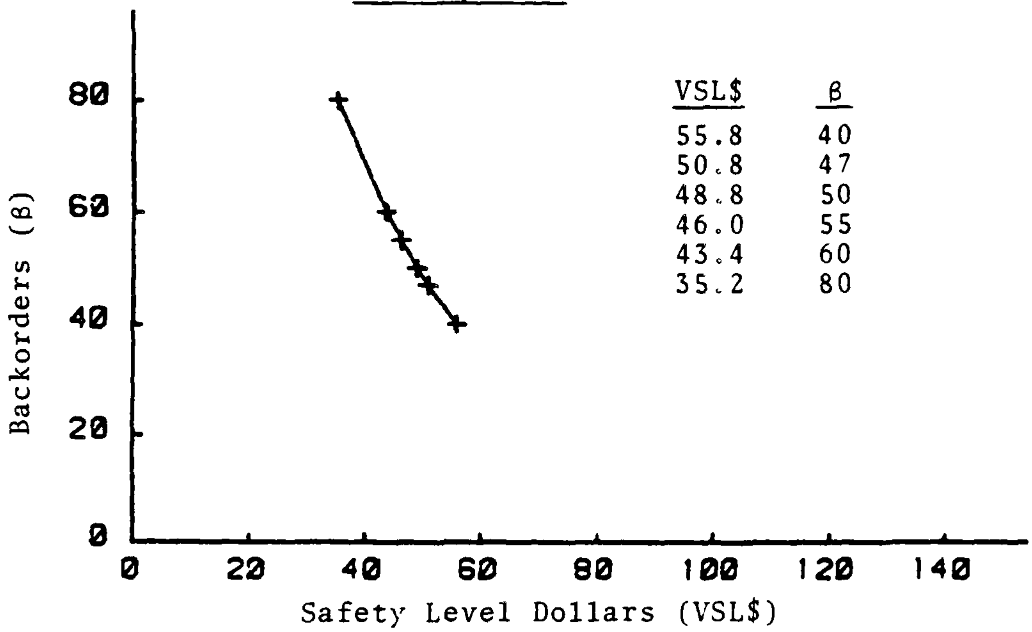
$$VSL\$ = C_1 \beta^{C_2}$$

which can be transformed into the linear regression model

$$\ln(VSL\$) = \ln(C1) + C2\ \ln(\beta)$$

Another model suggested by the form of the curve was

January 1976



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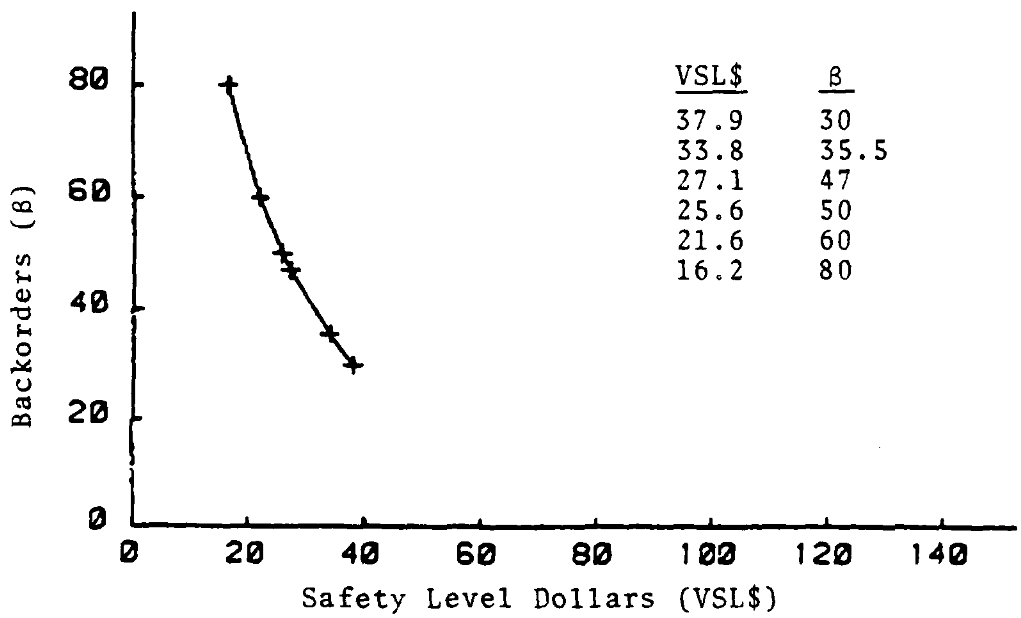


Figure 5

Two Examples of Actual VSL$-β Curves

$$VSL\$ = C1 + C2\ \ln(\beta)$$

To help determine the form of the curve, the DLA model was re-examined. If Model 3 is taken as a simplification of the DLA model, it can be further reduced. Recognizing that the SC is a constant during any period, Model 3 is transformed from

$$VSL\$ = C_0 + C_1 SC\ \ln(2.56\ \beta) + C_2 SC\ \ln(SC)$$

to its simplest form

Model 6: $VSL\$ = C_0 + C_1\ \ln(\beta)$

which is the function suggested above.

The procedure to test Model 6 was to first regress the six VSL$ values against the ln(β) values for each period to determine how well the curve fit in each period. These regressions also produced the coefficients C_0 and C_1 for each period. To determine whether C_0 and C_1 could be determined from the available indices, they were regressed against these variables. Then the regression equations for C_0 and C_1 were substituted into Model 6 and the resulting regression model was tested using the SPSS stepwise procedure which provides statistics indicating the accuracy of a regression model. These were also used to compare all of the models developed.

Forecasting Methods

Although the regression models indicate the relationships between VSL$, β, and the other indices in any period, the values of the indices are not known for the upcoming quarter which is the period of interest. Therefore, some method must be used to determine the new values of the indices which are included in the regression models before the quarterly update is run. Then these values can be included in the best regression model to determine the new VSL\$ - β curve from which the appropriate β can be selected to set safety levels in the quarterly update. The method selected was to test some basic forecasting techniques on each index and to select the most accurate technique for each.

Evaluating Forecasting Methods

The accuracy of a forecasting technique can be expressed in various terms which are measures of the average amount of error the technique produces for n forecasts, where the error in period t is e_t = actual value - forecast value. One term is the mean squared error,

$$MSE = \frac{1}{n} \sum_{t=1}^{n} e^2_t$$

Squaring the error terms prevents positive and negative errors from cancelling each other and, thus, results in a measure of the magnitude of the error expected in any period. However, the MSE is measured in the same units as the index being forecast. Therefore, when the index value varies significantly over time, the e^2_t in a period with a high value will be greater than the e^2_t in a period with the same relative error, but a low index value. The MSE indicates the average error over the periods measured, but the error in a particular period is biased by the index level. Another measure, expressed in terms of the actual error rather than the squared error, is the mean absolute deviation:

$$MAD = \frac{1}{n} \sum_{t=1}^{n} |e_t|$$

Like the MSE, the MAD is measured in the units of the index being forecast and, therefore, will be biased by the index level.

To avoid this problem, some measures of forecasting accuracy express the error as a percentage of the index value. One such measure is the mean percent error:

$$MPE = \frac{1}{n} \sum_{t=1}^{n} \frac{100e_t}{X_t}$$

where X_t is the actual value in period t. The MPE allows positive and negative errors to cancel each other. While this may be useful in measuring the cumulative effect of the errors, it does not measure the expected error for a specific period. Another measure which expresses the error as a percentage is the mean absolute percent error:

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \frac{100|e_t|}{X_t}$$

This is a measure of the expected amount of error produced by a forecasting technique which is not biased by the level of the index being forecast. Therefore, the MAPE was selected as the most appropriate measure to compare the accuracy of the forecasting techniques which were tested.

Moving Averages

Two of the techniques tested on each index in the regression models were simple moving averages and double moving averages. The method of simple moving averages forecasts the value of a variable by using the average value over past periods for the upcoming period. The number of periods averaged can range from one, which produces the so-called naive forecast, to any higher number, with the smoothing effect increasing as the number of periods increases. The smoothing effect reduces the effect on the forecast value of random variations from one period to the next. The technique of simple moving averages can be represented as follows:

$$S_{t+1} = \frac{1}{n}(X_t + X_{t-1} + \dots + X_{t-n+1})$$

where

S_t = forecast for period t

X_t = actual value for period t

n = number of periods included in the average

Simple moving averages can produce useful short-range forecasts when the variable tends to vary about a fixed value. However, the technique is not generally suitable when the data exhibits trend, seasonal, or cyclical patterns (30:34-35).

The technique of double moving averages is generally more accurate when these more complicated patterns exist. Essentially, this method recognizes that the simple moving average will lag behind trended data by some amount. In addition, a moving average of the simple moving average will lag behind the simple moving average by about the same amount. Thus, by adding the difference between the simple moving average and the double moving average back to the simple moving average, a more accurate forecast is produced. An n-period double moving average can be summarized as follows:

$$S''_{t+1} = S_{t+1} + \frac{n+1}{n-1}(S_{t+1} - S'_{t+1})$$

S'_{t+1} is the moving average of the simple moving average S_{t+1} and S''_{t+1} is the double moving average forecast. The term n+1/n-1 is an additional adjustment which has been found to make the technique more accurate. An important limitation of this technique is that 2N periods of data must be used, or twice the data required for simple averages (30:41-44). Both moving average techniques also require that the best number of periods to average be determined. The method used here was to calculate the MAPE using several different numbers of periods. Then the number selected was that which gave the lowest MAPE.

Exponential Smoothing

Two other techniques tested on the indices were single and double exponential smoothing. An advantage that these methods have over moving averages is that data from different periods can be given different weights. Single exponential smoothing gives some selected weight α to the most recent observed value and exponentially decreasing weights to older values. The general relationship is:

$$S_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + \alpha(1-\alpha)^2 X_{t-2} + \alpha(1-\alpha)^3 X_{t-3} + \ldots$$

This expression can be rewritten as:

$$S_{t+1} = \alpha X_t + (1-\alpha)S_t$$

While single exponential smoothing can provide slightly better forecasts than simple moving averages by given greater weight to the most recent data, it is also generally not suitable for data which has a pattern of basic changes over time (30: 36-39). However, as with moving averages, double smoothing can provide more accurate forecasts for data which has a trend pattern. The same concept underlying the double moving average explains the double exponential smoothing technique, which can be represented as:

$$S''_{t+1} = S_{t+1} + \frac{1}{1-\alpha}(S_{t+1} - S'_{t+1})$$

S''_{t+1} is the double exponential smoothing forecast and S'_{t+1} is the single exponentially smoothed S_{t+1}. Again, $\frac{1}{1-\alpha}$ is an additional adjustment which improves the accuracy of the forecast (30:44-47). Both exponential smoothing methods require that the best value of α be determined and this was again accomplished by calculating the MAPE for several values. Thus, α was selected based on the lowest MAPE.

Testing the Models

As mentioned earlier, the two most recent sets of data were reserved to test the models. After the models were developed and compared using statistical measures, the index values were forecast for these two periods using the best forecasting technique for each index. Then the forecast values were included in the models to calculate VSL$ based on the same values of β actually used in the two periods. The difference between the actual VSL$ in the period and the value of VSL$ calculated from a model represents the prediction error that would have resulted using the model in that period. These results were used as a final comparison of how well the models may be able to predict the VSL$ - β relationship for future periods.

Summary

This chapter defined the variables in the 21 sets of data used to develop the regression models. The six models were developed by examining the theoretical relationships between the variables in the DLA model and using the SPSS stepwise regression procedure to select the most significant variables. The SPSS statistics were also used as one means of evaluating how well the models might predict the VSL$ - β relationships. Four forecasting methods were then tested on each index required in a regression model and the best technique for a specific index was selected based on the lowest MAPE. Finally, the models and forecasting techniques were combined to predict a VSL$ value for the most recent two periods and the forecast errors were used to evaluate the predictive value of the models.

CHAPTER 4

RESULTS AND ANALYSIS

This chapter presents the results obtained by attempting to solve the research problem using the methodology presented in Chapter 3. First, the results of the regression analysis on the six VSL$ - β models are presented along with a discussion of the strengths and weaknesses of each model based on the regression statistics. Then, the results from applying each forecasting technique to each of the variables in the regression equations is presented and the best of the tested methods is selected for each variable. Finally, the predictive accuracy of each model is demonstrated by using the models to forecast the VSL$ - β relationship for two periods.

Regression Analysis

The results of the regression analysis for each group of models is presented first. This is followed by comparison of the results for the different models.

Linear Models

The potential independent variables for these two models were the available inventory indices. The correlation coefficients (r) and their significance (P) are given in

TABLE 1

Correlation Coefficients for the Linear Models

	β	SC	OLD SC	VSL$	DEMAND$	ITEMS	0-ITEMS
FREQ							
(r)	.0791	.4534	.3000	.3833	.5607	.9791	.2002
(P)	.367	.019	.093	.043	.004	.001	.192
0-ITEMS							
(r)	.6205	.3832	.4308	.2066	.2882	.2679	
(P)	.001	.043	.026	.184	.103	.120	
ITEMS							
(r)	.1340	.4850	.3376	.4063	.5671		
(P)	.281	.013	.067	.034	.004		
DEMAND$							
(r)	.1999	.9753	.8146	.9574			
(P)	.192	.001	.001	.001			
SL							
(r)	-.2248	-.0273	-.0238	.1666			
(P)	.164	.453	.459	.235			
VSL$							
(r)	.1343	.9587	.7955				
(P)	.281	.001	.001				
OLD SC							
(r)	.6217	.8697					
(P)	.001	.001					
SC							
(r)	.3270						
(P)	.074						

Table 1. This shows that β does not have a strong relationship with any individual variable. The highest correlation is with the OLD SC, which explains only about 39 percent (the value of r^2) of the variation in β. VSL$, however, is strongly related with both SC and DEMAND$. The correlation between VSL$ and OLD SC is also fairly strong and the

significance of the three correlations is high. Therefore, a fairly accurate linear model was expected using VSL$ as the dependent variable, while Model 2, with β as the dependent variable, was not expected to fit the data well.

The regression equation derived for Model 1 was:

$$VSL\$ = 9.2817 + .5895SC - .0004\beta + .2084(OLD\ SC)$$

Table 2 gives a summary of the regression analysis for Model 1.

TABLE 2

Regression Summary for Model 1

Step	Variable Entered	r^2	Overall F	Significance
1	SC	.91902	215.619	< .001
2	β	.95499	190.956	< .001
3	OLD SC	.96244	145.195	< .001

At step 3, the F values of each of the variables not already in the equation fell below 2.0 and, therefore, they were not entered. DEMAND$ may have been expected to enter the equation because of its high correlation with VSL$. However, SC entered the equation first and it is also highly correlated with DEMAND$. Therefore, DEMAND$ could explain little of the remaining variation in VSL$ which caused the partial F value for DEMAND$ to be lower than the partial F for both β and OLD SC. An analysis of the coefficients in this model is shown in Table 3.

TABLE 3

Analysis of Model 1 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	2.997	9.591	.007
C_1	0.087	53.413	< .001
C_2	0.00009	16.929	.001
C_3	0.114	3.371	.084

Table 3 shows that the coefficients have a high significance except for C_3, the coefficient for OLD SC. The high r^2 and high overall significance of Model 1 indicate that the model fits the data quite well. However, the low significance of C_3 reduces confidence in this model somewhat because this indicates that old SC may not have an important influence on VSL$ after accounting for the influence of β and SC.

The regression analysis for Model 2 produced the following equation:

$$\beta = -4888.5256 + 710.296\,(\text{OLD SC}) - 632.226\,\text{VSL\\$} + 1.364(\text{O ITEMS})$$

Table 4 provides a summary of the regression analysis for this model. As expected, by the low correlation coefficients, Model 2 does not fit the data very well. Although the overall significance of the model is high, the r^2 is relatively low. This indicates a high probability that the independent variables contribute to determining the value of VSL$, but

TABLE 4

Regression Summary for Model 2

Step	Variable Entered	r^2	Overall F	Significance
1	OLD SC	.38644	11.972	.003
2	VSL$	.74008	25.626	< .001
3	0-ITEMS	.80295	23.091	< .001

TABLE 5

Analysis of Model 2 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	9068.121	0.290	.597
C_1	118.289	36.057	< .001
C_2	132.546	22.752	< .001
C_3	0.586	5.424	.032

that only about 80 percent of the VSL$ variation is explained by the model. Table 5 also shows that the significance of C_3, the coefficient of 0-ITEMS, is not especially high, which also reduces confidence in the model. Overall, Model 2 showed little promise of defining the VSL$ - β relationship with the desired accuracy and was rejected after examining the results of the regression analysis.

Regressions Based on the DLA Model

The first regression model derived from analyzing the

DLA safety level model was Model 3:

$$VSL\$ = C_0 + C_1 SC\ \ln(\beta) + C_2 SC\ \ln(SC)$$

This model uses the current system constant in all terms. The correlation coefficients for the terms in this model are shown in Table 6 along with the coefficients which apply to Models 4 and 5. This table only gives the correlations between variables which are in the same model, as the other relationships are of no interest.

TABLE 6

Correlation Coefficients for Models 3, 4, and 5

	VSL$	SC ln(OLD SC)	(OLD SC) ln(β)	SC ln(SC)
SC ln(β)				
(r)	.9365	.9969		.9948
(P)	.001	.001		.001
SC ln(OLD SC)				
(r)	.9498			
(P)	.001			
(OLD SC)ln(β)				
(r)	.7562			
(P)	.001			
(OLD SC)ln(OLD SC)				
(r)	.8032		.9960	
(P)	.001		.001	
SC ln(SC)				
(r)	.9612			
(P)	.001			

Because the term SC ln(SC) has a higher correlation with VSL$, it would normally be the first variable entered in the regression equation for the model by the SPSS stepwise subprogram. However, this may have prevented SC ln(β) from entering the equation because of its high correlation with SC ln(SC). Since β is required in the equation to define the VSL\$ - β relationship, SC ln(β) was forced into the regression model first. Surprisingly, even with the high correlation between SC ln(β) and SC ln(SC), the partial F value for SC ln(SC) after SC ln(β) entered the equation was 39.43. Therefore, SC ln(SC) was also entered and the resulting regression model was

$$VSL\$ = 27.067 - .124SC\ \ln(\beta) + .382SC\ \ln(SC)$$

A summary of the regression analysis for Model 3 is given in Table 7 and an analysis of the model coefficients is shown in Table 8.

TABLE 7

Regression Summary for Model 3

Step	Variable Entered	r^2	Overall F	Significance
1	SC ln(β)	.87712	135.616	< .001
2	SC ln(SC)	.96148	224.669	< .001

This model fits the data very well as shown by the high r^2. The high overall F value shows that the probability that VSL\$ is not related to the variables in the model is extremely low. Likewise, the analysis of the coefficients shows that they are highly significant with relatively small standard

TABLE 8

Analysis of Model 3 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	0.0296	17.549	.001
C_1	0.0609	39.429	< .001
C_2	4.0967	43.652	< .001

deviations. These results indicate that Model 3 may provide good predictions of VSL$ with less than four percent of the variation in VSL$ unexplained by the model.

Model 4 is the same as Model 3, except that ln(OLD SC) is substituted for ln(SC). The form of the model is:

$$\text{VSL\$} = C_0 + C_1 SC \ln(\beta) + C_2 SC \ln(\text{OLD SC})$$

SC ln(β) was again forced into the regression equation first to ensure that β was in the model. The regression equation resulting from Model 4 was:

$$\text{VSL\$} = 21.410 - 0.108 SC \ln(\beta) + [illegible]63 SC \ln(\text{OLD SC})$$

The regression summary for this model is shown in Table 9, and the analysis of the coefficients is given in Table 10. Although the significance of this model is very high, the value of r^2 leaves over eight percent of the variation in VSL$ unexplained. In addition, the standard deviations of the coefficients are relatively large and the significance of C_1 is not high. This model is inferior to Model 3 in

TABLE 9

Regression Summary for Model 4

Step	Variable Entered	r^2	Overall F	Significance
1	SC ln(β)	.87712	135.616	< .001
2	SC ln(OLD SC)	.91901	.02.128	< .001

TABLE 10

Analysis of Model 4 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	6.0745	12.422	.002
C_1	0.0555	3.7753	.068
C_2	0.1189	9.3119	.007

these respects and still requires that the current system constant be known. Therefore, this model was rejected at this point in favor of Model 3.

The purpose of Model 5 was to substitute OLD SC for SC to avoid the requirement to know the current system constant. This model had the general form

$$VSL\$ = C_0 + C_1(\text{OLD SC})\ \ln(\beta) + C_2(\text{OLD SC})\ \ln(\text{OLD SC})$$

Once again, it was necessary to force the first variable in the equation first because it had a lower correlation with VSL$ than the second variable. The regression equation for this model was:

TABLE 11

Regression Summary for Model 5

Step	Variable Entered	r^2	Overall F	Significance
1	(OLD SC)ln(β)	.57178	25.369	< .001
2	(OLD SC)ln(OLD SC)	.88400	68.585	

TABLE 12

Analysis of Model 5 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	7.2929	48.306	< .001
C_1	0.0650	37.072	< .001
C_2	0.1419	48.448	< .001

$$\text{VSL\$} = 50.687 - 0.402(\text{OLD SC})\ln(\beta) + 0.988(\text{OLD SC})\ln(\text{OLD SC})$$

The regression summary and analysis of the coefficients are given in Tables 11 and 12. This model has a high overall significance and there is no problem with the significance or standard deviation of any coefficient. However, like Model 4, the value of r^2 is low, meaning that a large percentage of the variation in VSL$ is left unexplained by the model. This appears to indicate that the current system constant is required to accurately relate VSL$ and β.

Model Based on the VSL$ - β Curve

The first step in analyzing Model 6 was to test the relationship between VSL$ and $\ln(\beta)$ for each period using the regression model:

$$VSL\$ = C_0 + C_1 \ln(\beta)$$

These results are given in Appendix B, which shows the value of C_0, C_1, and the r^2 for each period. The average r^2 for the 21 periods is greater than 0.998, showing that the data fits this model almost perfectly. The next step in the analysis of this model was to determine whether C_0 and C_1 were dependent on the available indices. The correlations between C_0, C_1, and the indices are given in Table 13. The correlations between the other variables were included in Table 1.

TABLE 13

Correlation Coefficients for Model 6

		SC	OLD SC	DEMAND$
C_0	(r)	.9673	.9351	.9152
	(P)	.001	.001	.001
C_1	(r)	.9749	.9083	.9362
	(P)	.001	.001	.001

		ITEMS	0-ITEMS	FREQ	C_1
C_0	(r)	.3547	.4431	.3131	.9939
	(P)	.057	.022	.084	.001
C_1	(r)	.3559	.4076	.3216	
	(P)	.057	.033	.078	

C_0 and C_1 were then regressed against the indices to derive the following equations:

$$C_0 = 31.399 + 1.988SC + 1.086(OLD\ SC) - 21.797FREQ$$

$$C_1 = 9.125 + 0.451SC + 0.121(OLD\ SC) - 4.539FREQ$$

The regression summaries for these two equations are given in Tables 14 and 15, which show that C_0 and C_1 can be determined very accurately given the necessary indices. The analysis of the coefficients is not presented because the purpose was not to be able to actually determine C_0 and C_1, but to find the indices which should be substituted for them in Model 6. The coefficients in the final form of Model 6 were derived by a separate regression analysis.

TABLE 14

Regression Summary for C_0

Step	Variable Entered	r^2	Overall F	Significance
1	SC	.93560	276.046	< .001
2	OLD SC	.97175	309.634	< .001
3	FREQ	.98223	313.271	< .001

TABLE 15

Regression Summary for C_1

Step	Variable Entered	r^2	Overall F	Significance
1	SC	.95040	364.091	< .001
2	FREQ	.96866	278.195	< .001
3	OLD SC	.97781	249.697	< .001

Substituting the appropriate indices for C_0 and C_1 in Model 6 gives the following general model:

$$VSL\$ = (SC+(OLD\ SC)+FREQ) + (SC+(OLD\ SC)+FREQ)\ln(\beta)$$

This was transformed into the following regression model:

$$VSL\$ = C_0 + C_1SC + C_2(OLD\ SC) + C_3FREQ + C_4SC\ \ln(\beta) + C_5(OLD\ SC)\ \ln(\beta) + C_6FREQ\ \ln(\beta)$$

This was the final form for Model 6. The SPSS regression analysis produced the following equation:

$$VSL\$ = 25.581 + 7.271SC - 103.019FREQ - .621SC\ \ln(\beta) + 9.106FREQ\ \ln(\beta)$$

The regression summary and the analysis of the regression coefficients for Model 6 are presented in Tables 16 and 17.

TABLE 16

Regression Summary for Model 6

Step	Variable Entered	r^2	Overall F	Significance
1	SC	.91902	215.619	< .001
2	SC ln(β)	.95322	183.385	< .001
3	FREQ	.96300	147.506	< .001
4	FREQ ln (β)	.97400	149.822	< .001

The high r^2 shows that this model fits the data very well, with less than three percent of the VSL$ variation unexplained. Model 6 also has a very high overall significance and the significance of the individual coefficients is relatively

TABLE 17

Analysis of Model 6 Coefficients

Coefficient	Standard Error	Partial F	Significance
C_0	7.9708	10.300	.005
C_1	1.9639	13.706	.002
C_3	37.5957	7.509	.015
C_4	0.1870	11.031	.004
C_6	3.5017	6.763	.019

high. Overall, this model could be expected to determine VSL$ very accurately, given the values of the independent variables.

Summary of Regression Analyses

Model 1 was the better of the two linear models, with an r^2 of better than .962. Model 2 was rejected because it fit the data much worse than Model 1. Model 3 was the best of those derived from the DLA model, with an r^2 higher than .961. Both Models 4 and 5 were rejected because of their comparatively low r^2 values. Model 6 had the highest r^2 of .974. Of the three models which had a close fit with the data, Model 3 had the most significant coefficients. The significance of one coefficient in Model 1 was relatively low and all the coefficients in Model 6 were less significant than those in Model 3, although the significance of each was still fairly high. Each of these three were considered good models, which were further tested by using forecasts of the variables to predict VSL$.

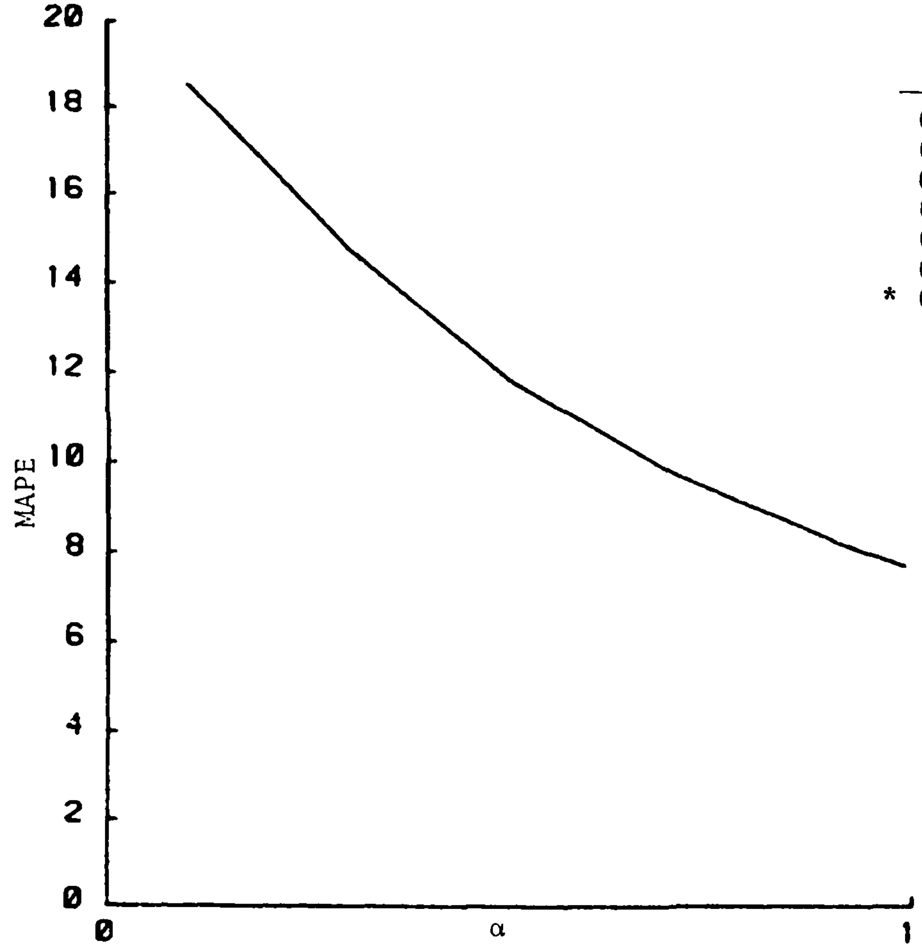
Forecasting Results

Models 1, 3, and 6 have a total of four independent variables amongst them: SC, β, OLD SC, and FREQ. However, OLD SC is always known and β will be set based on its relationship with VSL$. Therefore, it was necessary to forecast only the two unknown variables, SC and FREQ. The initial forecasting analysis included data from all of the available 21 periods.

Figure 6 shows the MAPEs which result from using single exponential smoothing with varying values of α to forecast SC. Figure 7 shows the values of MAPE using double exponential smoothing to forecast SC. Figure 8 gives the MAPE values which resulted from forecasting SC using single moving averages and Figure 9 gives the MAPEs resulting from using double moving averages. In these figures, the best MAPE value for each method is marked by an asterisk, while the best overall MAPE for the system constant is marked with a double asterisk. Figures 10 through 13 give the same results for FREQ. These figures show that the lowest MAPE for the system constant resulted from using double exponential smoothing and an α of 0.60. The lowest MAPE for FREQ came from using a double moving average over ten periods.

The forecasts which result from using these two forecasting models are plotted against the actual values in Figures 14 and 15. The period numbers correspond with those in Appendix A. Figure 14 shows that the forecasts for SC, using double exponential smoothing with $\alpha = .6$, follow the actual values very closely except during periods six and eight. The large errors in these two periods were obviously caused by the extreme drop in periods six and seven, followed by the sharp increase in period eight. According to DESC, the very low SC in periods six and seven were the result of a policy decision which was expected to significantly lower the SC for these periods (1). Because of this, these two periods were regarded as distortions of the normal SC trend, which could influence the selection of the best forecasting method or the best α or number of periods. In Figure 15, FREQ shows the same distortion in these two periods, which again was caused by the policy change. Because the two periods did not represent the normal operating policy and they significantly differ from the other periods, it was decided that they should not be used in comparing the forecasting methods.

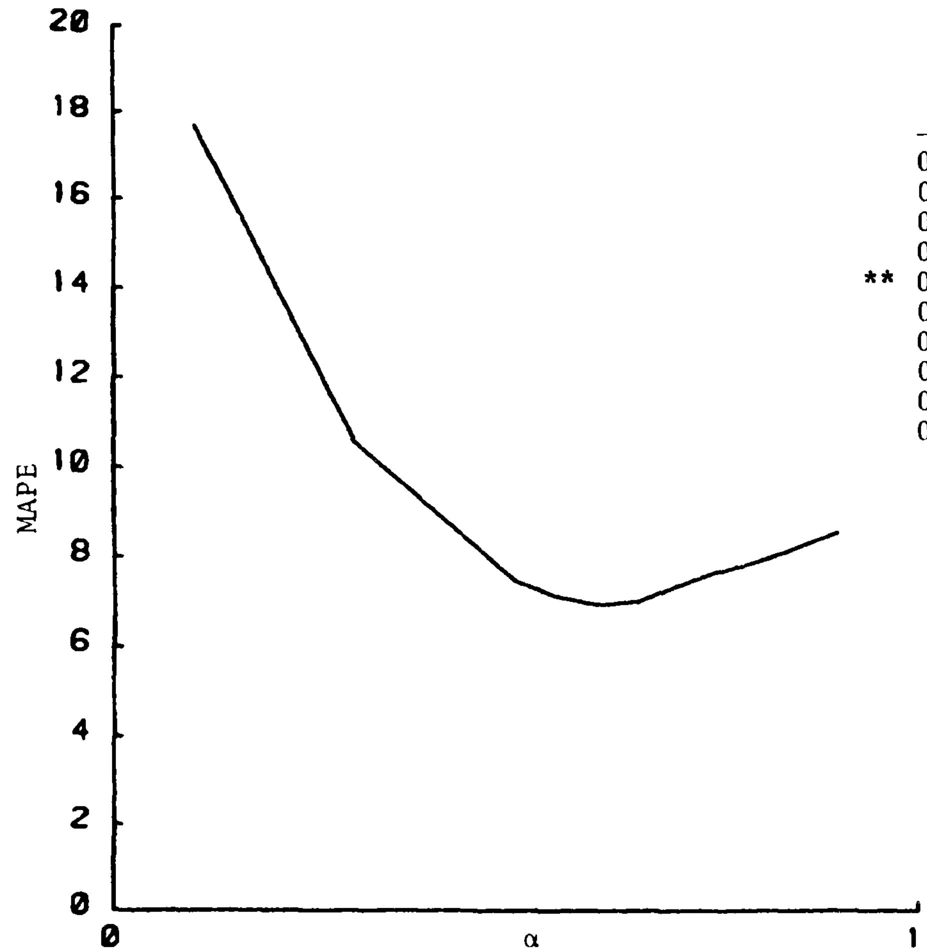
Therefore, each of the methods was again tested on both SC and FREQ, using only the data from periods 8 through 21. Figures 16 through 19 show the new MAPEs for SC, and Figures 20 through 23 give the new results for FREQ. The figures show that double exponential smoothing with an α of 0.80 produced the lowest MAPE for both SC and FREQ. The new low MAPE for SC of 3.17 is lower than the previous MAPE of 6.92, as should be expected after eliminating the large errors caused by periods 6 and 7. The new low MAPE of .95 for FREQ is not quite as low as the previous of .92.



α	MAPE
0.1	18.531
0.3	14.738
0.5	11.802
0.7	9.763
0.9	8.264
0.95	7.942
* 0.99	7.696

Figure 6

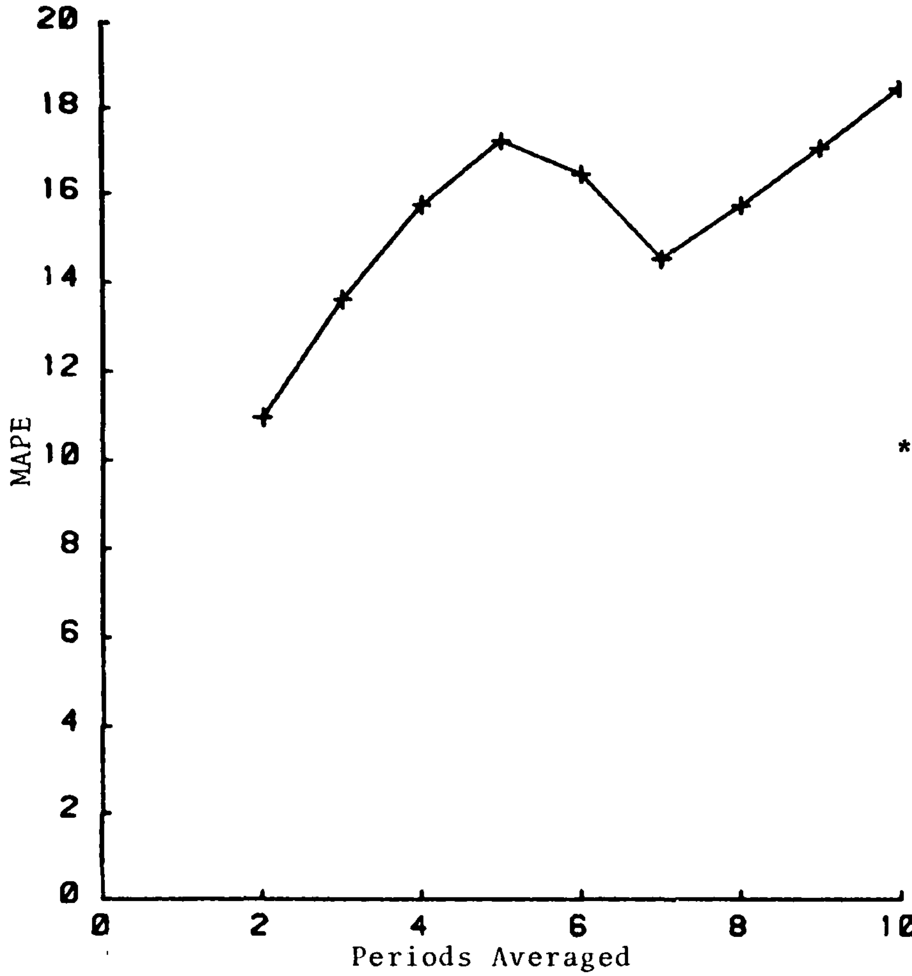
Single Exponential Smoothing Errors for System Constant (Periods 1-21)



	α	MAPE
	0.1	17.681
	0.3	10.555
	0.5	7.444
	0.55	7.106
**	0.6	6.919
	0.65	6.987
	0.7	7.349
	0.75	7.649
	0.8	7.896
	0.9	8.534

Figure 7

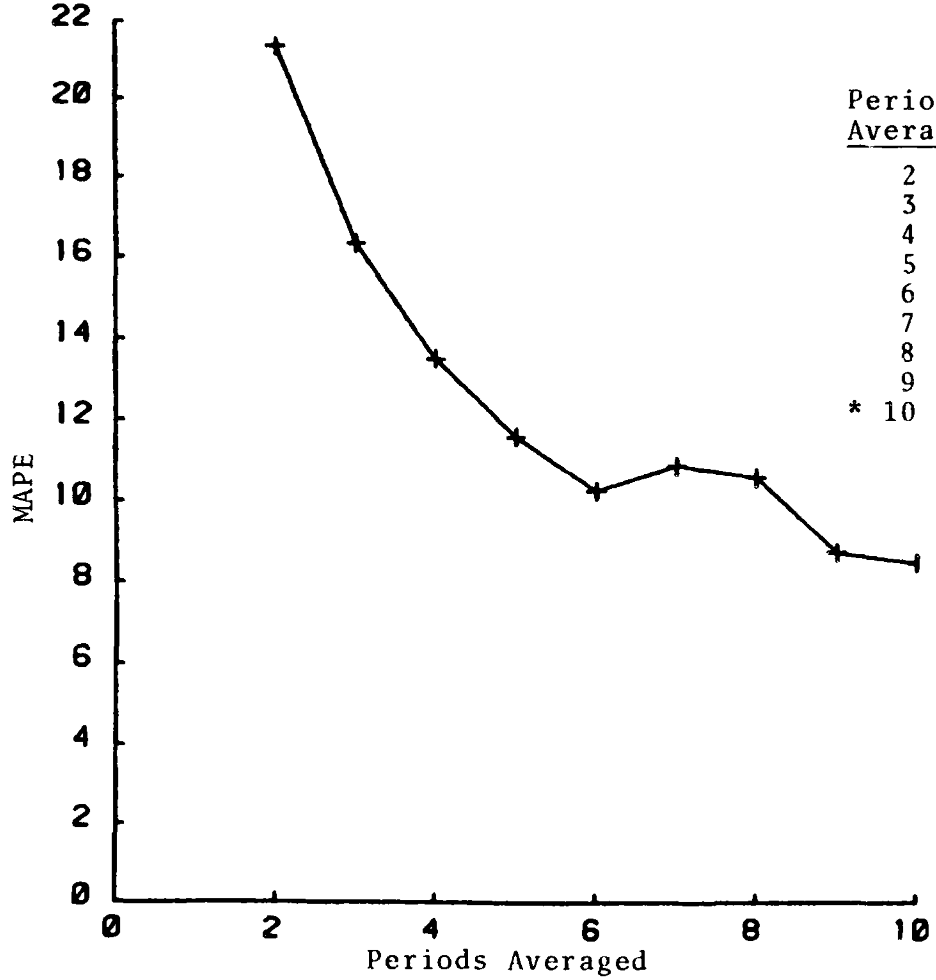
Double Exponential Smoothing Errors for System Constant (Periods 1-21)



Periods Averaged	MAPE
* 2	10.95
3	13.59
4	15.72
5	17.20
6	16.41
7	14.49
8	15.70
9	17.02
10	18.44

Figure 8

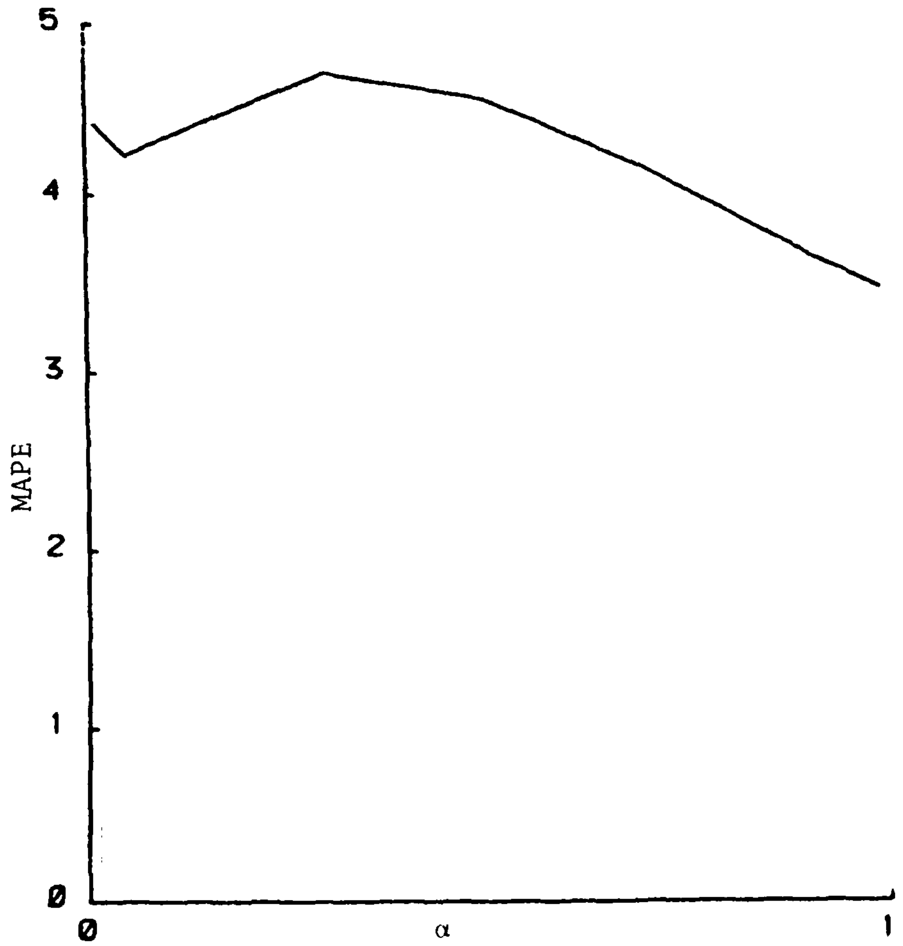
Single Moving Average Errors for
System Constant (Periods 1-21)



Periods Averaged	MAPE
2	21.37
3	16.34
4	13.48
5	11.56
6	10.24
7	10.85
8	10.57
9	8.73
* 10	8.48

Figure 9

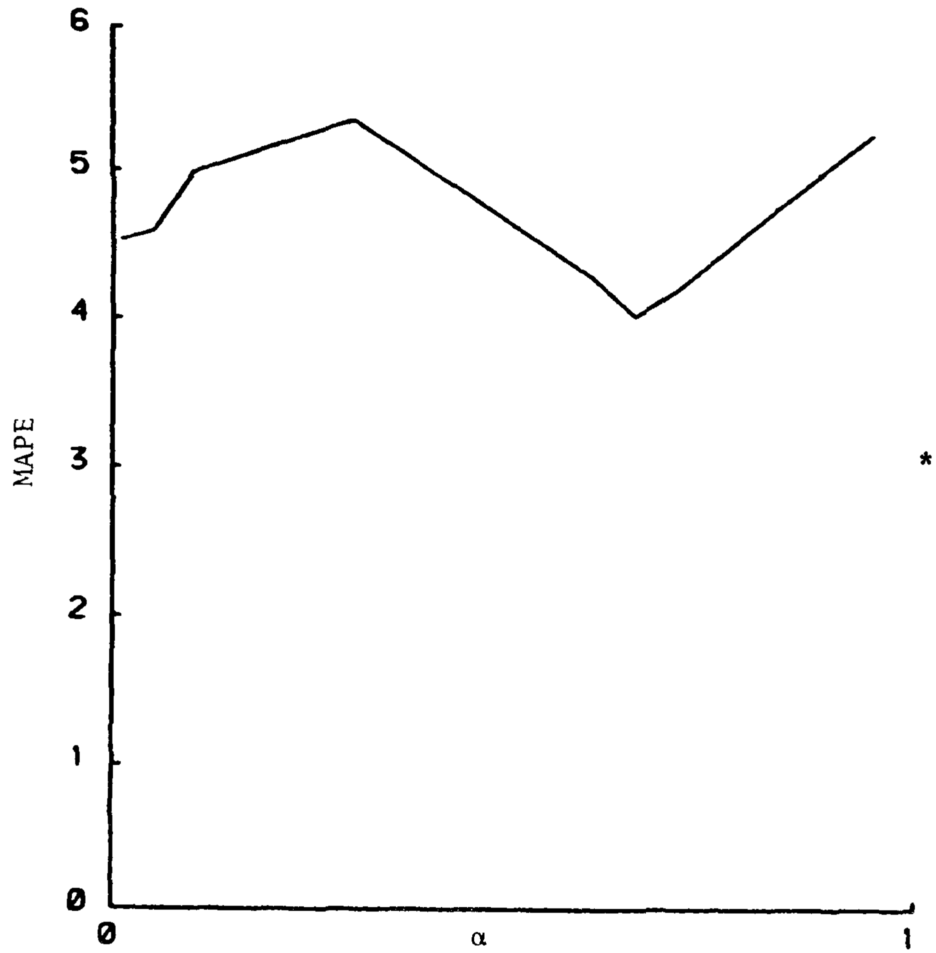
Double Moving Average Errors for System Constant (Periods 1-21)



α	MAPE
0.01	4.416
0.05	4.225
0.1	4.332
0.3	4.699
0.5	4.535
0.7	4.126
0.9	3.644
0.95	3.545
* 0.99	3.459

Figure 10

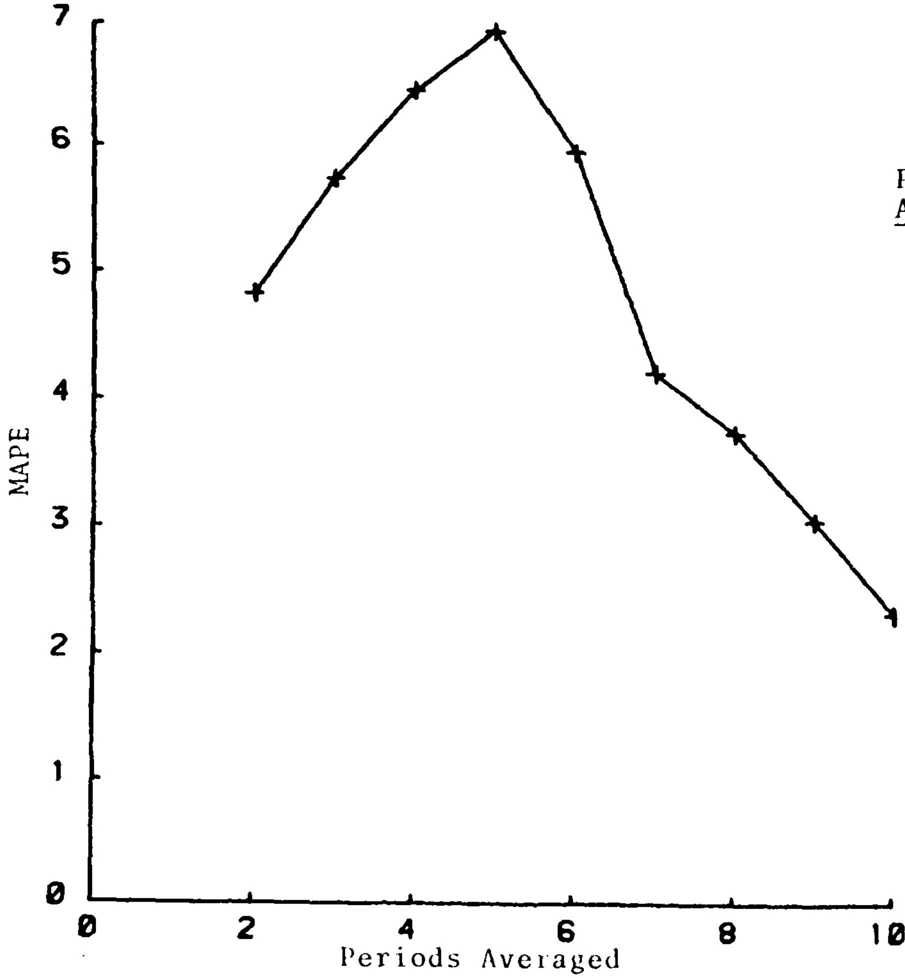
Single Exponential Smoothing Errors for Frequency (Periods 1-21)



α	MAPE
0.01	4.528
0.05	4.582
0.1	4.985
0.3	5.345
0.5	4.613
0.6	4.257
* 0.65	4.006
0.7	4.168
0.9	5.042
0.95	5.247

Figure 11

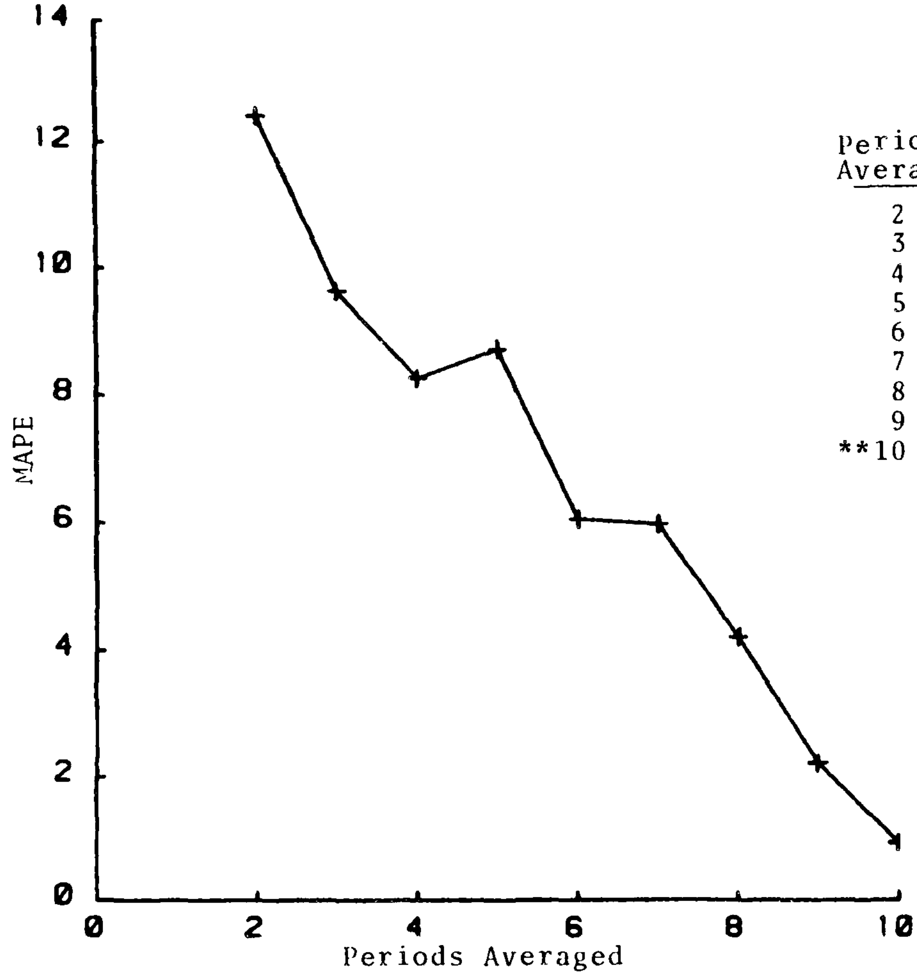
Double Exponential Smoothing Errors
for Frequency (Periods 1-21)



Periods Averaged	MAPE
2	4.82
3	5.73
4	6.45
5	6.94
6	5.94
7	4.20
8	3.73
9	3.05
*10	2.32

Figure 12

Single Moving Average Errors
for Frequency (Periods 1-21)



Periods Averaged	MAPE
2	12.42
3	9.61
4	8.25
5	8.70
6	6.05
7	5.97
8	4.21
9	2.22
**10	0.92

Figure 13

Double Moving Average Errors
for Frequency (Periods 1-21)

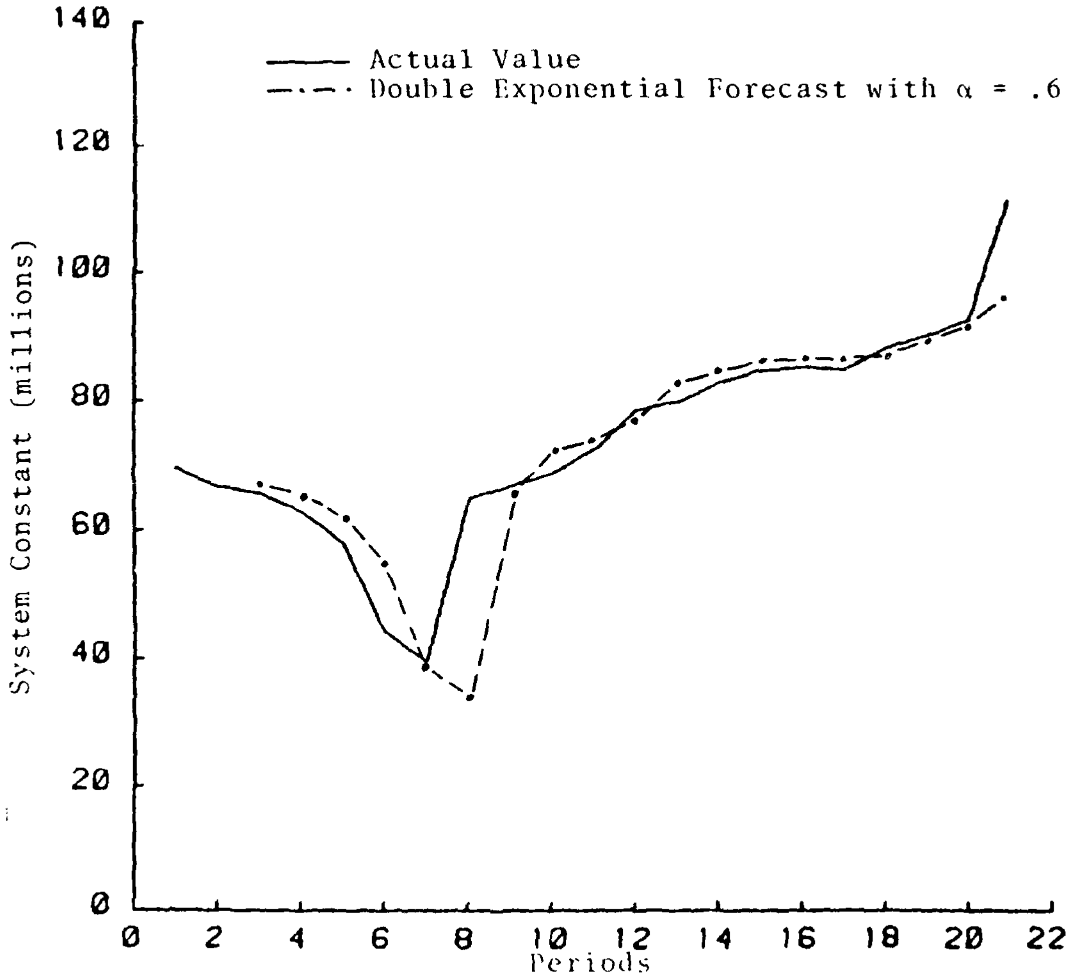


Figure 14

Actual System Constant Versus Best Forecast Method (Periods 1-21)

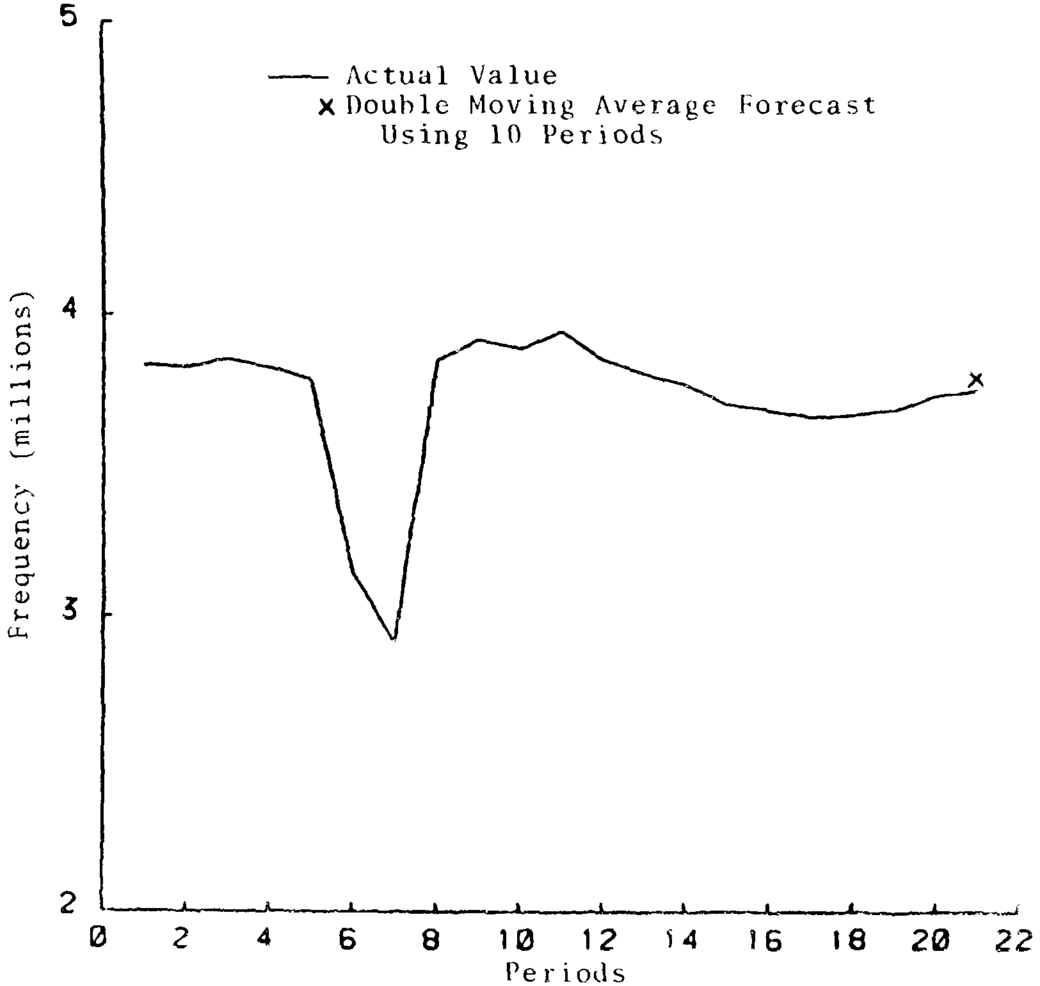


Figure 15

Actual Frequency Versus Best Forecast Method
(Periods 1-21)

AD-A110 970 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL--ETC F/G 5/1
RELATING EXPECTED INVENTORY BACKORDERS TO SAFETY STOCK INVESTME--ETC(U)
SEP 81 D M CARPENTER
UNCLASSIFIED AFIT-LSSR-91-81 NL

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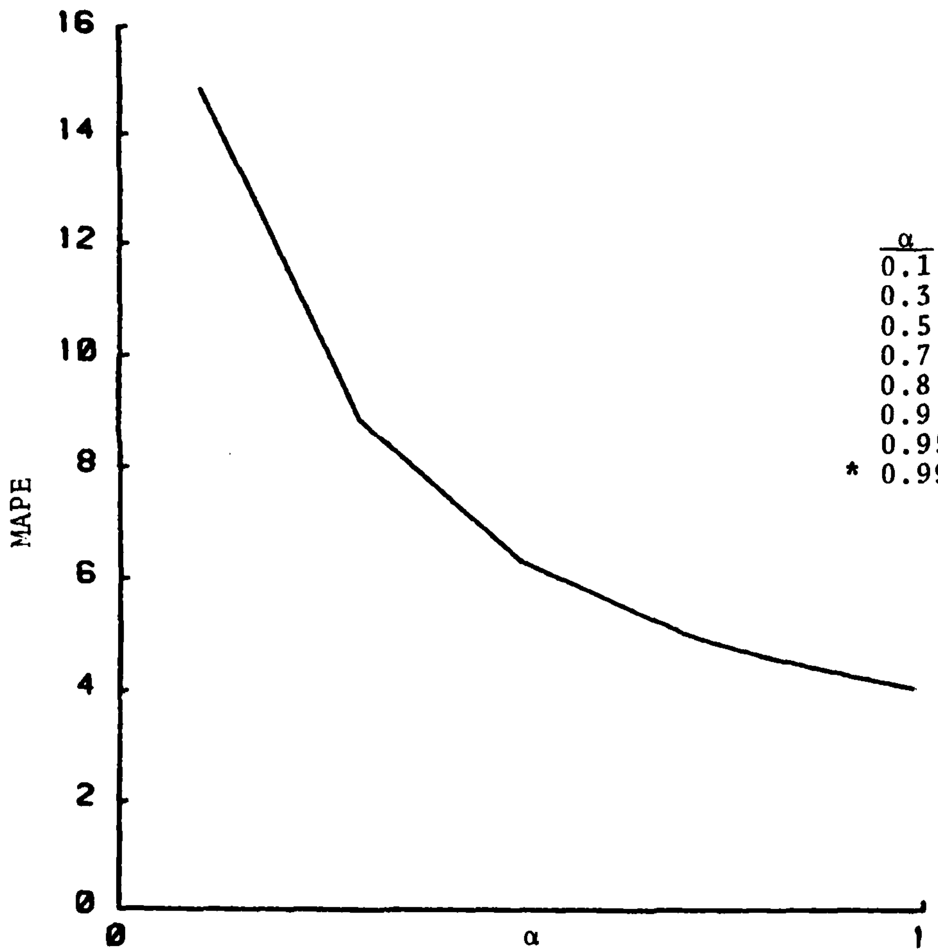
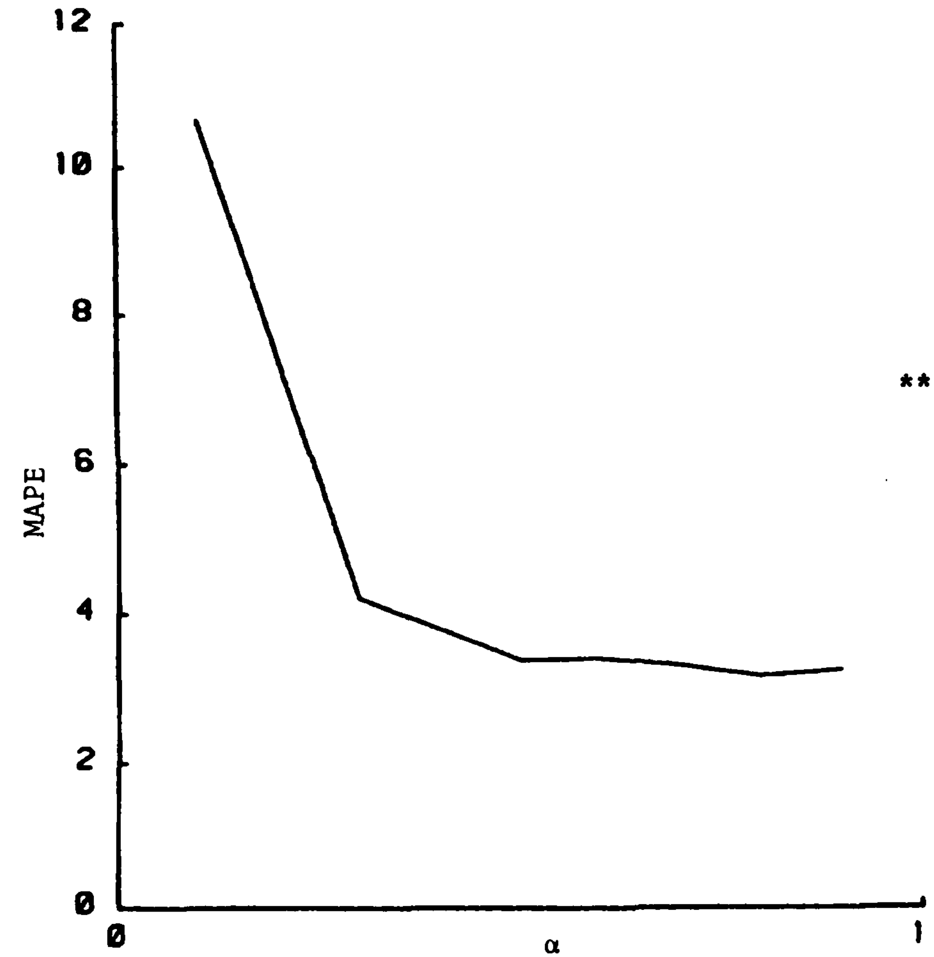


α	MAPE
0.1	14.854
0.3	8.828
0.5	6.321
0.7	5.041
0.8	4.633
0.9	4.314
0.95	4.174
* 0.99	4.070

Figure 16

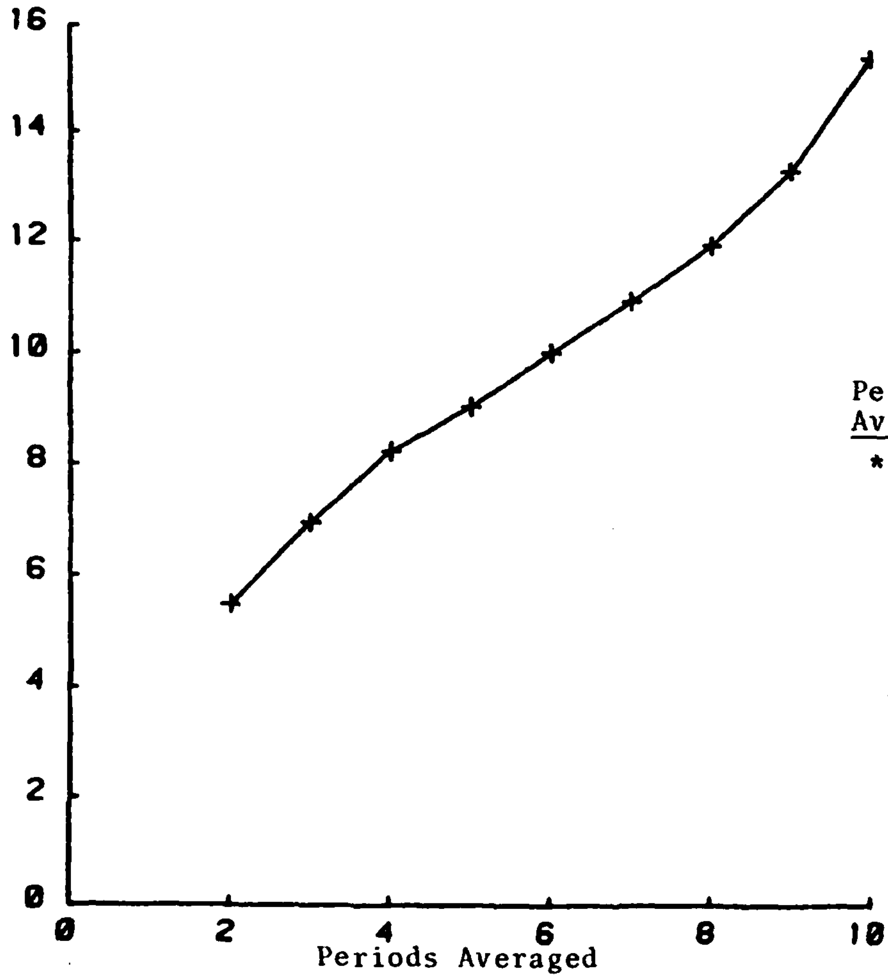
Single Exponential Smoothing Errors
for System Constant (Periods 8-21)



	α	MAPE
	0.1	10.654
	0.3	4.210
	0.5	3.386
	0.6	3.407
	0.7	3.307
	0.75	3.234
**	0.8	3.171
	0.85	3.202
	0.9	3.248

Figure 17

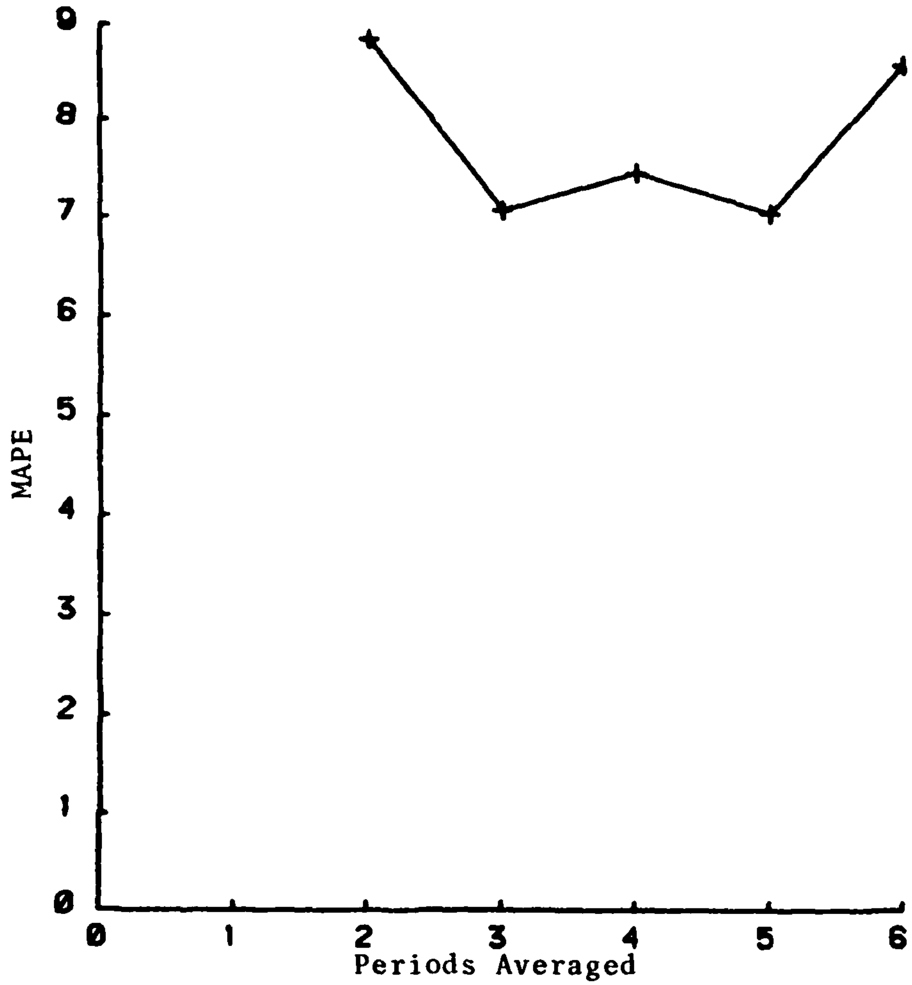
Double Exponential Smoothing Errors
for System Constant (Periods 8-21)



Periods Averaged	MAPE
* 2	5.49
3	6.95
4	8.22
5	9.01
6	9.98
7	10.90
8	11.90
9	13.26
10	15.38

Figure 18

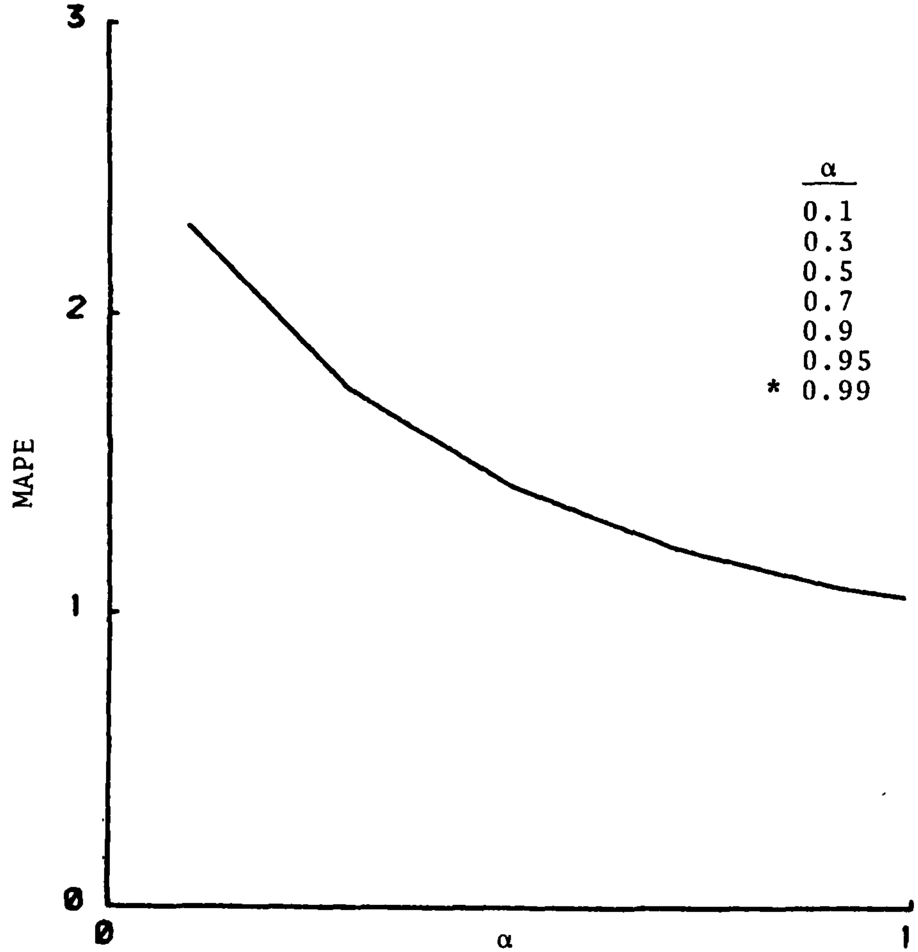
Single Moving Average Errors
for System Constant (Periods 8-21)



Periods Averaged	MAPE
2	8.85
3	7.06
4	7.44
* 5	7.03
6	8.58

Figure 19

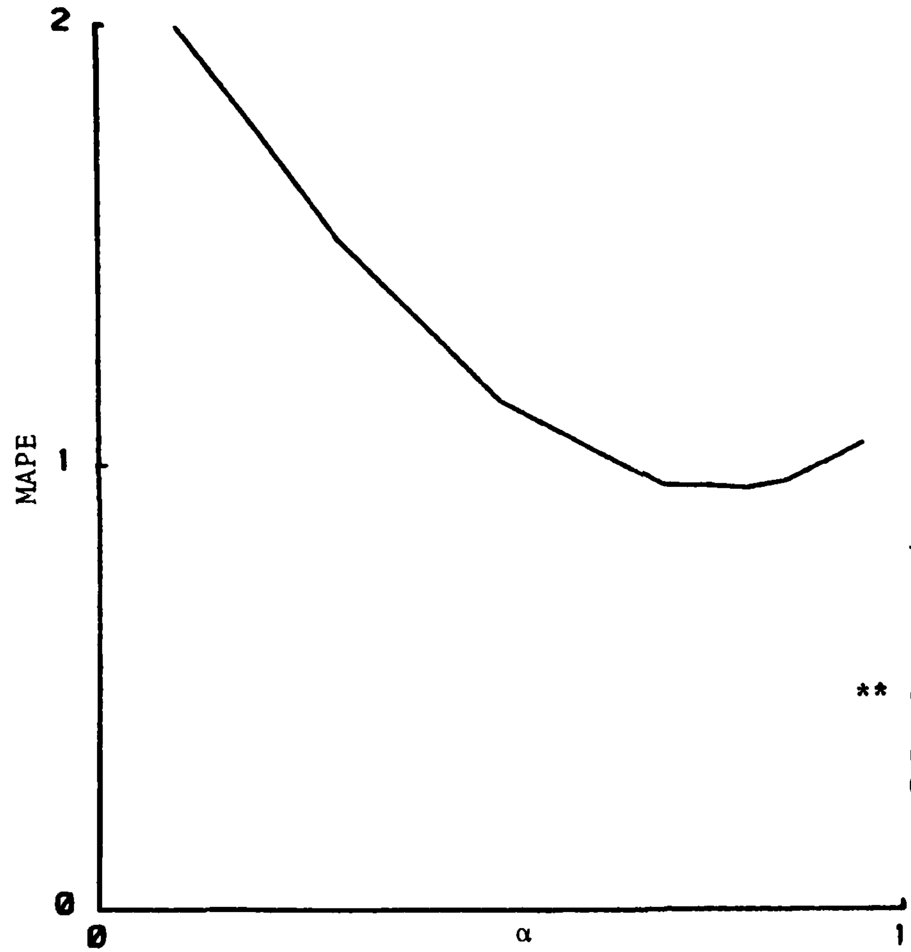
Double Moving Average Errors for System Constant (Periods 8-21)



	α	MAPE
	0.1	2.296
	0.3	1.748
	0.5	1.426
	0.7	1.222
	0.9	1.093
	0.95	1.073
*	0.99	1.059

Figure 20

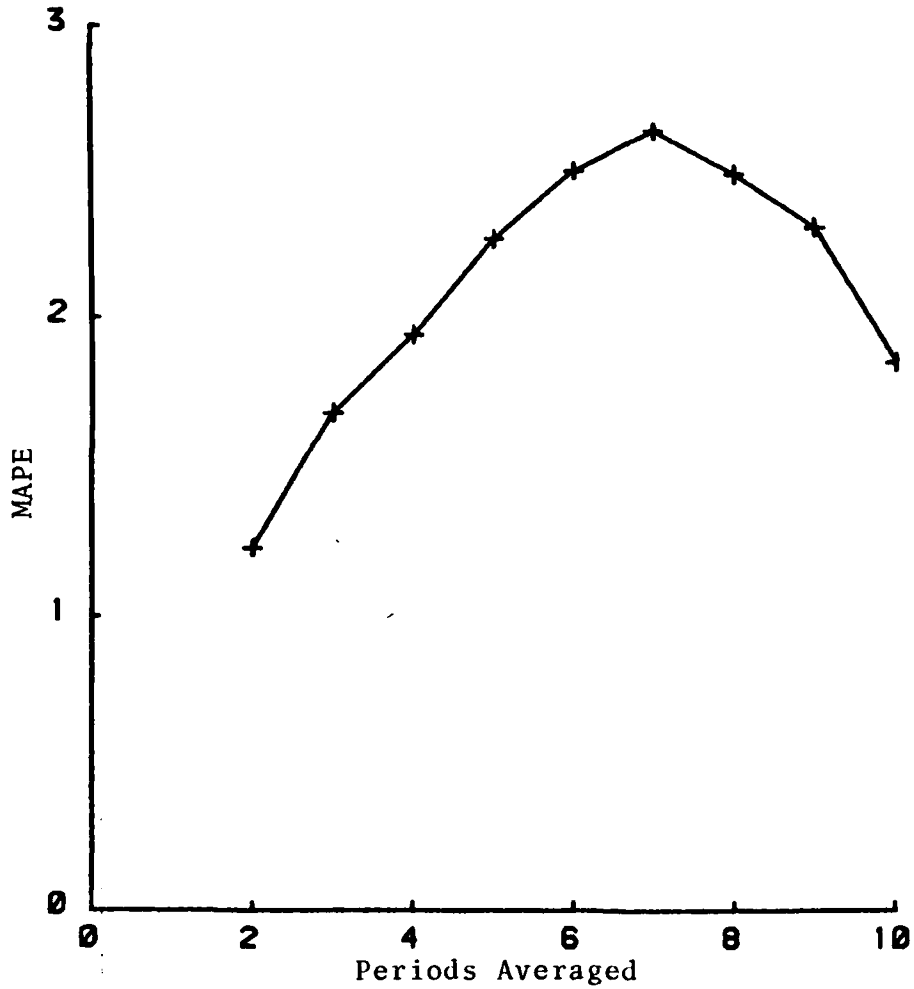
Single Exponential Smoothing Errors for Frequency (Periods 8-21)



	α	MAPE
	0.1	1.995
	0.3	1.499
	0.7	1.141
	0.75	0.955
**	0.8	0.953
	0.85	0.948
	0.9	1.004
	0.95	1.049

Figure 21

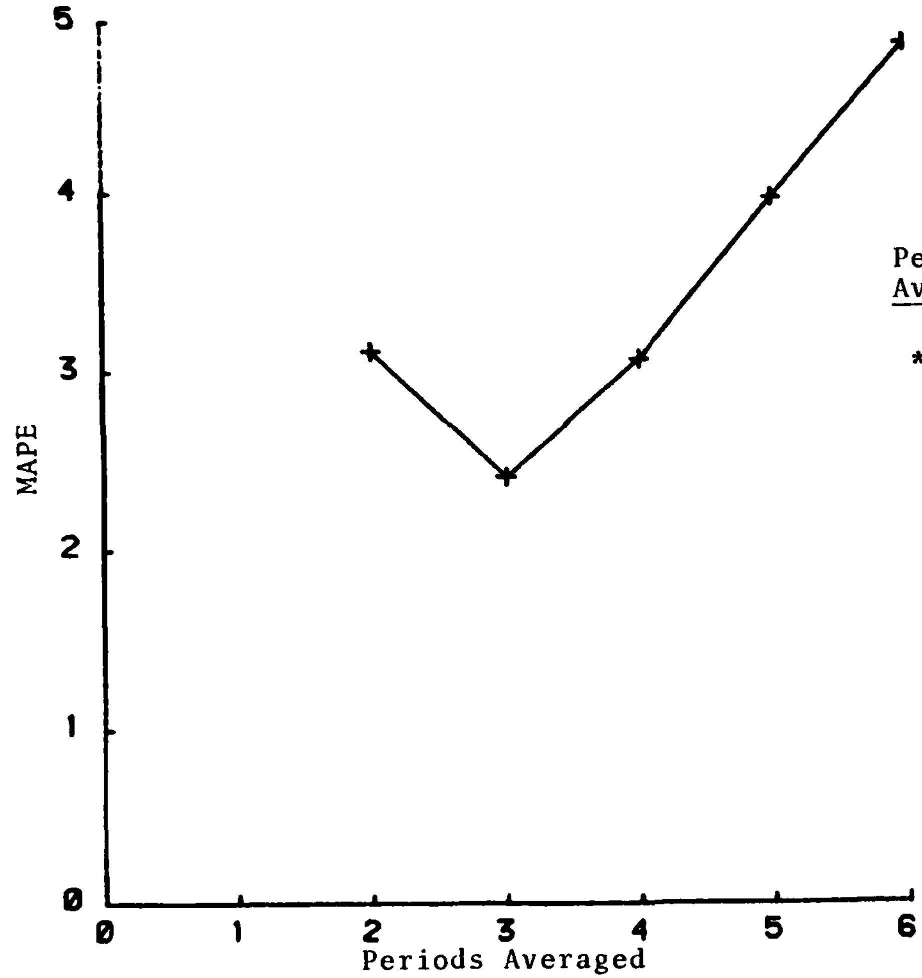
Double Exponential Smoothing Errors
for Frequency (Periods 8-21)



Periods Averaged	MAPE
* 2	1.23
3	1.68
4	1.94
5	2.26
6	2.49
7	2.63
8	2.48
9	2.30
10	1.85

Figure 22

Single Moving Average Errors
for Frequency (Periods 8-21)



Periods Averaged	MAPE
2	3.11
* 3	2.41
4	3.06
5	3.96
6	4.86

Figure 23

Double Moving Average Errors
for Frequency (Periods 8-21)

However, there was very little confidence in the previous results not only due to the two distorted periods, but also because the MAPE was based on a single forecast in period 21. There was only one period forecast, because the double moving average method averaging over ten periods requires 20 periods of data. Therefore, double exponential smoothing was selected as the best of the methods tested to forecast both SC and FREQ. The forecasts resulting from this method, with an α of 0.80, are plotted against the actual values of the indices in Figures 24 and 25. These show that the forecasts closely follow the actual values.

Model Predictions

Using double exponential smoothing with an α of 0.80 to forecast SC results in forecasts of 123.945 and 118.841 for periods 22 (March 1981) and 23 (June 1981). The absolute percent errors of these forecasts are 10.04 for period 22 and 3.23 for period 23. The large error for period 22 was caused by the sharp increase in period 21. The relatively small error in period 23 shows that the forecasts adjust quickly to large changes due to the high α. The FREQ forecasts for periods 22 and 23 are 3.776 and 3.752 with absolute percent errors of 0.80 and 0.79 respectively.

These forecasts were substituted into Models 1, 3, and 6, and VSL$ was calculated for each model based on the β used in both periods 22 and 23. The model predictions for period 22 are shown in Table 18 with the error for each model.

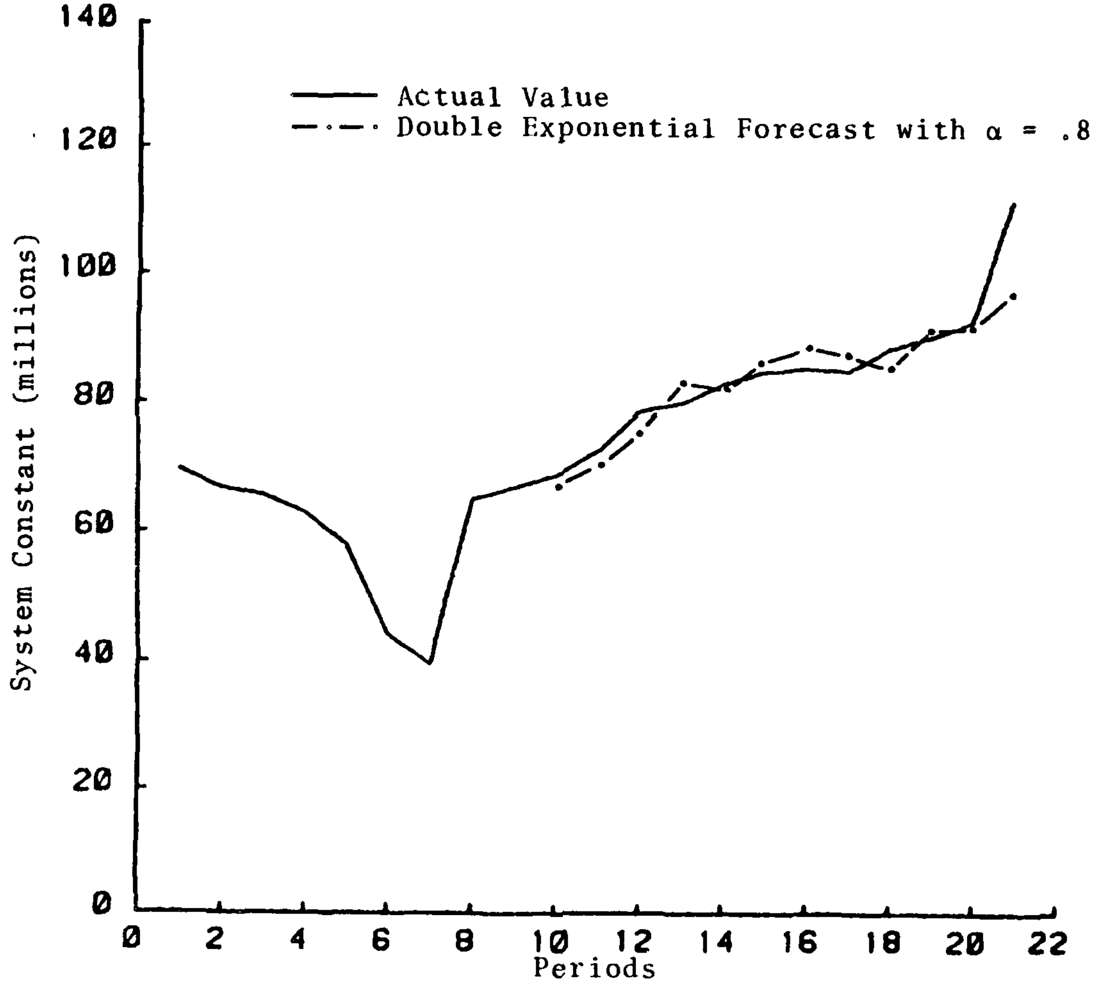


Figure 24

Actual System Constant Versus Best Forecast Method (Periods 8-21)

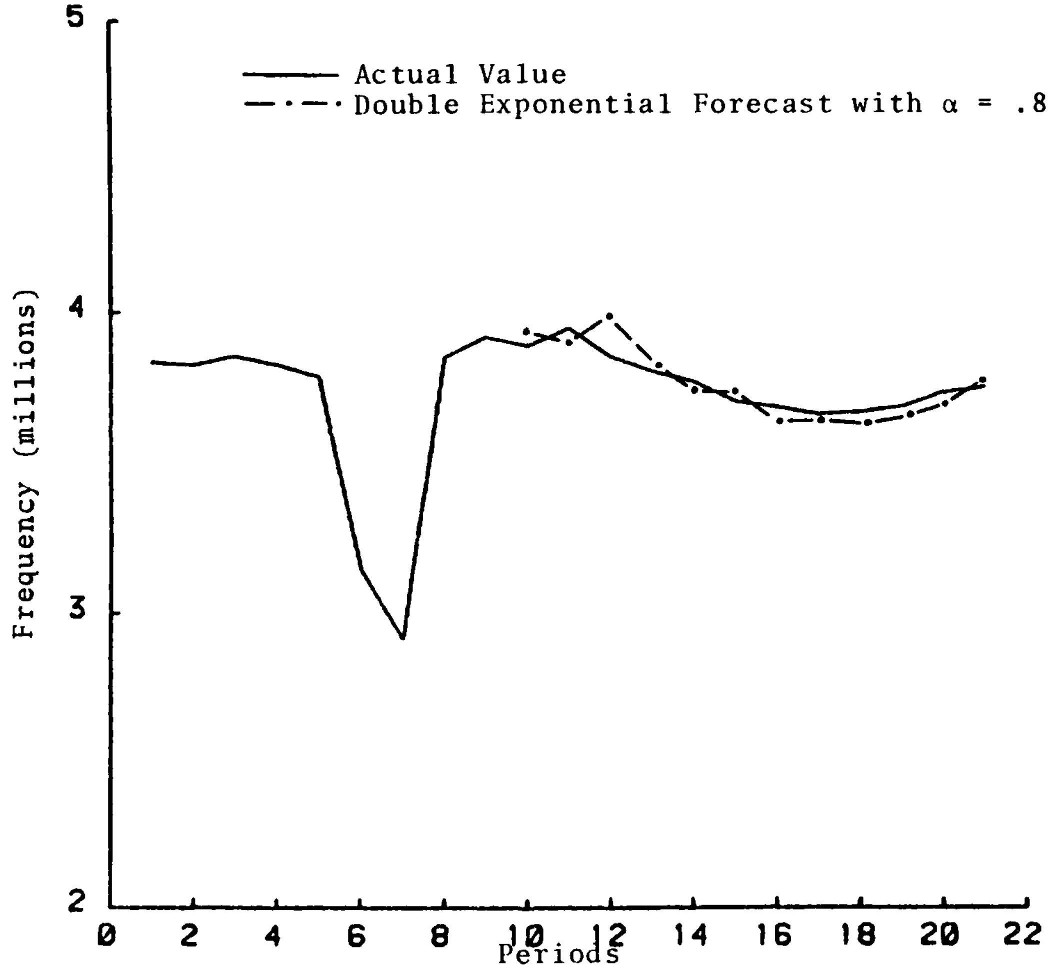


Figure 25

Actual Frequency Versus Best
Forecast Method (Periods 8-21)

TABLE 18

Model Predictions for Period 22
(β = 43,000)

Model	Actual VSL$	Predicted VSL$	Absolute Error	% Error
1	82.16	88.41	6.25	7.61
3	82.16	91.75	9.59	11.67
6	82.16	83.44	1.28	1.56

TABLE 19

Model Predictions for Period 23
(β = 35,000)

Model	Actual VSL$	Predicted VSL$	Absolute Error	% Error
1	92.59	88.81	3.78	4.08
3	92.59	89.79	2.80	3.02
6	92.59	88.45	4.14	4.17

The results for period 23 are given in Table 19. Model 6 has the lowest error in period 22 and the highest error in period 23. Model 3 produced the opposite results, with the highest error in period 22 and the lowest in period 23. However, the error for Model 6 in period 23 is still relatively low and close to the errors for the other two models, while the Model 3 error in period 22 is very large. Model 1 did not produce the best or the worst predictions for either period and appears to react slowly to changes.

The slow reaction to changes by Model 1 could be due to the OLD SC term in the model, which would cause the predictions to lag behind the actual values. Model 3, which is dominated by the SC, appears to react very quickly to changes in SC and perhaps overreact when the changes are large. Model 6 includes the FREQ index, which seems relatively more stable than the SC. This appears to somewhat dampen the effect of large changes in SC, although the model still seems to react quickly to SC changes. These results show that none of the models is clearly superior in all circumstances, and that their relative performance is affected by the magnitude of the index forecast errors.

Summary

Of the six regression models tested, three were rejected because they fit the data poorly. The three remaining models had high r^2 values, and each was used to predict VSL$ for two periods using forecast values for the required indices. The results showed that the two indices required could be forecast fairly accurately, except when there is a significant change in an index trend. The results of the model predictions were mixed with the best prediction apparently being determined by the nature of the changes in the indices. However, with one exception, the cumulative errors from the forecasts and the regression models were quite small.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Based upon the results presented in Chapter 4, it appears that when the independent variables are known, a regression model can define the VSL$ - β relationship fairly accurately. With the three best models, 96 to 97 percent of the variation in VSL$, during the 21 periods in the data base, could be explained by β and the other independent variables.

Using one of these models will eliminate the requirement to calculate several points on the VSL$ - β curve directly from the current data. Instead, the points can be calculated from the regression equation.

The choice of which regression equation to use is not completely clear based solely on the results presented in Chapter 4, because these did not show one model to be substantially more accurate than the others in all instances. Model 1, however, has more weaknesses than either Models 3 or 6. First, there is some question about whether Model 1 actually describes causal relationships between the variables or is simply the linear equation which fits the historical data the closest. In developing this model, there was no underlying hypothesis about the relationships or even the variables to be included in the model. In addition, the r^2

for Model 1 was not as high as the r^2 for Model 6, and one of the coefficients had a relatively low significance. Finally, Model 1 did not produce the best prediction for either of the two test periods. Based on these considerations, Model 1 appears to be inferior to both Models 3 and 6.

Both Models 3 and 6 were derived from the theoretical relationships between the variables. This provides high confidence that the models define a causal relationship between VSL\$, β, and the other variables, which will hold true for future periods. Although either model might serve to define the VSL\$ - β relationship, Model 6 has some advantages over Model 3. One important difference between the two is that Model 6 had the highest r^2. Model 6 also had a lower significance for its coefficients than Model 3; however, the significance levels were still high enough to have a very high confidence that the variables were responsible for determining VSL\$. Perhaps the most important difference between these two models was in the prediction errors for the two test periods. While both models had the highest error in one period and the lowest in the other period, Model 6 had a much lower average error because Model 3 greatly overreacted in period 22. This caused Model 3 to have an average error for the two periods of 7.345 percent, while Model 6 had an average error of 3.015 percent. Both the higher percent error of Model 3 and its apparent tendency to overreact to changes in SC were considered important drawbacks. Therefore, Model 6 is recommended as the model to use to define the VSL\$ - β relationship.

The values of SC and FREQ which are needed in Model 6 can either be forecasted or obtained directly from the current data. The results showed that FREQ can be forecasted with an average error of less than one percent. The forecasts for SC were also very accurate except in period 21, when there was a sharp increase in the actual value. The error was also large in period 22, while the forecast model was adjusting to the new SC level. It is the opinion of this writer that such large changes in the SC level could be anticipated by DESC personnel. In these periods when the SC forecasts will produce unacceptable errors, the value of SC should be determined directly from the current data either by calculating the value for all items or using a representative sample. During all other periods, the double exponential smoothing forecasts should be accurate enough to use in Model 6 to select the desired level for VSL$ and β.

The results of this research indicate that the cost of various levels of inventory performance can be found through aggregate measures of the inventory characteristics using a causal regression model and well-known forecasting techniques. While the research concentrated on the DESC inventory using the DLA model to set safety stock levels, the results may be of interest to other DoD components which also use the general DoD safety level model. Other organizations responsible for large inventories may also find some value in the general technique used here and the indication from these results that the relationship between cost and performance can be predicted with a fair degree of accuracy.

APPENDIX A

HISTORICAL DATA

Period	Quarter Beginning	β	SC (millions)	OLD SC (millions)	VSL$ (millions)
1	Jan 76	50,000	$69.60	$76.41	$48.86
2	Apr 76	40,000	66.74	69.60	46.68
3	Jul 76	40,000	65.63	66.74	44.62
4	Oct 76	40,000	62.92	65.63	43.27
5	Jan 77	39,000	58.03	62.92	40.44
6	Apr 77	35,500	44.26	58.03	33.78
7	Jul 77	23,410	39.70	44.26	34.30
8	Oct 77	20,000	64.90	39.70	47.01
9	Jan 78	25,500	66.78	64.90	49.88
10	Apr 78	25,100	68.82	66.78	51.30
11	Jul 78	26,100	72.57	68.82	55.06
12	Oct 78	26,100	78.58	72.57	61.57
13	Jan 79	35,000	79.95	78.58	58.07
14	Apr 79	38,000	82.90	79.95	59.99

Period	DEMAND$ (millions)	ITEMS	0-ITEMS	FREQ (millions)
1	$256.54	157,937	18,733	3.834
2	253.46	156,161	15,062	3.826
3	245.78	160,742	17,286	3.853
4	239.00	156,596	16,201	3.825
5	214.48	154,312	16,710	3.785
6	174.67	105,345	15,922	3.147
7	143.51	105,731	14,258	2.918
8	260,23	155,153	16,620	3.849
9	270.40	159,195	15,664	3.915
10	275.75	157,562	15,296	3.886
11	288.36	165,032	18,296	3.943
12	309.71	153,853	14,714	3.851
13	310.31	152,686	13,661	3.803
14	315.20	149,992	17,254	3.769

Period	Quarter Beginning	β	SC (millions)	OLD SC (millions)	VSL$ (millions)
15	Jul 79	39,400	$84.82	$82.90	$59.07
16	Oct 79	35,000	85.32	84.82	59.38
17	Jan 80	39,000	85.00	85.32	65.22
18	Apr 80	46,000	88.28	85.00	59.60
19	Jul 80	43,000	90.25	88.28	65.00
20	Oct 80	48,200	92.64	90.25	57.68
21	Jan 81	32,800	111.63	92.64	84.60
22	Apr 81	43,000	112.64	111.63	82.16
23	Jul 81	35,000	115.12	112.64	92.59

Period	DEMAND$ (millions)	ITEMS	0-ITEMS	FREQ (millions)
15	$311.68	150,694	18,170	3.705
16	314.14	148,230	15,795	3.686
17	325.21	147,695	16,630	3.662
18	305.91	148,232	19,376	3.668
19	314.89	151,431	18,936	3.687
20	327.50	151,541	20,468	3.735
21	391.87	151,720	16,008	3.750
22	390.62	151.613	17,156	3.746
23	396.98	150,627	13,999	3.724

APPENDIX B

RESULTS FROM REGRESSING VSL$ AGAINST ln(β)

Period	C_0	C_1	r^2
1	165.00	-29.70	.9994
2	154.60	-29.20	.9979
3	152.04	-29.08	.9969
4	148.28	-28.44	.9975
5	139.60	-27.00	.9967
6	113.40	-22.30	.9966
7	90.98	-18.20	.9949
8	124.50	-26.17	.9940
9	143.09	-28.75	.9975
10	146.64	-29.56	.9975
11	162.86	-33.04	.9999
12	182.48	-37.06	.9999
13	187.53	-36.41	.9998
14	210.01	-41.26	.9999
15	212.14	-41.65	.9999
16	203.88	-40.60	.9997
17	224.53	-43.47	.9999
18	223.97	-42.92	.9999
19	233.90	-44.90	.9999
20	227.37	-43.78	.9999
21	265.05	-51.75	.9997

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